

Version: A

# Examination in Microeconomics A

## Summer Term 2008

### Handling of the exam

- Please check carefully whether your exam sheets are complete and correct, objections after the exam cannot be considered:
  - There are 2 **versions** of this exam, which are denoted by A and C respectively. Please check carefully, whether the version on the question sheet corresponds to the one on the solution sheet.
  - The **question sheet** (including the pages with the general remarks) consists of 8 pages. In addition there is a **solution sheet**, which consists of 3 pages.
- The use of resources other than a non-programmable calculator and at most one dictionary is not allowed. The use of other resources (e.g. programmable calculators, your own concept paper) leads to the disqualification from the exam.
- You have 120 minutes to solve the exam.
- The **exam** consists of 5 True- / False questions, each consisting of 5 subquestions, and 3 Text Problems again each consisting of 5 subquestions.
- For the True- / False- questions you have to decide whether a statement is true or false. For *each* subquestion you have to mark on the solution sheet whether the statement is true (T) or false (F). You will be awarded points according to the following rule: If your answer is correct, you will obtain *3 points* per statement. If your answer is wrong or if both answers are marked, you will obtain *0 points*. If no answer is given, then you will get *1 point*. For the True- / False- questions you can therefore obtain at most obtain 75 points.
- The **Text Problems** have, on the one hand, Multiple-Choice-subquestions (MC) with 5 answers provided for each question, where *exactly one of these answers is correct*. On the other hand, there are numerical subquestions (N), where you have to fill in a number on the solution sheet in encoded form. For each subquestion you get 5 points if answered correctly and 0 otherwise. For the Text Problems you can therefore at most obtain 75 points. Here is an example on how to encode integers in the numerical subquestions: Suppose the solution to the question is **503**. Then this number has to be filled in as follows:

Zahl Frage	100er	10er	1er
1	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
2	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
3	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>
4	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
5	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
6	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
7	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
8	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
9	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
0	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>

Figure 1:

**Important:** Mark the zero in the first column if the solution is a two-digit number. Similarly, mark the zero in the first and in the second column if the solution is a single-digit number.

- You will have passed the exam with certainty, if you obtain at least *75 points* or if you are among the 75% best participants of the exam.

Handling of the solution sheet:

- You **only** have to submit the solution sheet at the end of the exam. Answers on concept sheets or on the question sheet will not be considered. We recommend that you fill in the solutions at the **end of the exam** in order to avoid corrections. Please start to fill in your answers **at least 5 minutes before the end of the exam**. The supervisors have orders to collect the solution sheets, even if you have not yet filled in your answers.
- *Please fill in the whole circle, do not mark answers with a cross! Only unambiguously legible solutions can yield points. Please do not use TippiEx to correct your answers!*
- You must sign your solution sheet at the bottom, otherwise your exam is not **valid**.
- If you do not wish that we publish your registration number, your points and your expected mark on our homepage, you have to mark the respective field on the solution sheet. If you mark this field, you have to wait for your grade until it is announced by the "Studienbüro", which may take some time.

Concerning the content of the exam

1. Assume that the "Ceteris-Paribus" condition holds. This means that all variables that are not explicitly changed remain constant. If we ask for example about the repercussions of the change of one variable (e.g.  $p_1$ ), you have to assume that the other variables (e.g.  $p_2$ ) remain constant, unless explicitly stated otherwise.
2. If we say that a variable (e.g.  $p_1$ ) is changed, we mean a marginal change, which is strictly different from zero, unless explicitly stated otherwise.
3. Assume infinitely divisible goods, unless explicitly stated otherwise.
4. Assume strictly positive and finite prices and income.
5. Assume that consumers maximize their utility and firms maximize profit.
6. Market demand functions are always weakly decreasing, market supply functions are weakly increasing.

Good luck!

# 1 True-/False questions

1.1 A consumer's preferences can be represented by the utility function  $u(x_1, x_2) = x_1^{0.4} x_2^{0.6}$ . Her income is  $Y > 0$ . Consider a decrease of the first good's price from  $p_1$  to  $p'_1 < p_1$  (the price  $p_2$  does not change). Which of the following statements are true?

- a The demand for good 1 increases.
- b Let  $Y'$  be the income such that the consumer is just able, at prices  $(p'_1, p_2)$ , to afford the bundle that she purchased at prices  $(p_1, p_2)$ . Statement:  $Y' < Y$ .
- c Let  $x_1^S$  be the quantity that the consumer purchases at price  $p'_1$ , given that she has income  $Y'$ . Statement: The difference in the quantities  $x_1^S - x_1$  due to the substitution effect is positive.
- d The change in quantities  $x_1^S - x_1$  caused by the substitution effect is independent of  $Y$ .
- e Let  $x'_1$  be the quantity of good 1 that the consumer purchases at price  $p'_1$  (and income  $Y$ ). Statement: The difference in quantities  $x'_1 - x_1^S$  due to the income effect is positive.

1.2 A firm produces a homogenous good with the production function  $Y = AK^\alpha$ , where  $A > 0$ ,  $0 < \alpha < 1$  are constants and  $K > 0$  is the capital stock used in production. There are no fixed costs. The market interest rate (price of capital) is  $r > 0$ , the price of the produced products is  $p > 0$ . Which of the following statements are true?

- a For each arbitrary level of  $K$ , the firm's profit is given by  $pAK^\alpha - rK$ .
- b The optimal capital stock is a linearly decreasing function of the interest rate  $r$ .
- c At the optimum, the firm's profit is  $(1 - \alpha)pAK^\alpha$ .
- d If the firm maximizes its profit, then its (total) payment to the creditors is an increasing function of the interest rate  $r$ .
- e The firm's cost function is convex.

1.3 A consumer plans his consumption for 2 periods  $t=1,2$ . In the two periods his income is given by  $Y_1$  and  $Y_2$  respectively. The consumer may save or borrow at the same interest rate  $r > 0$ . His utility is  $U(c_1, c_2) = u(c_1) + u(c_2)$ , where  $u$  is a strictly increasing and strictly concave function.

- a The consumer's budget constraint is  $c_1 + c_2 = Y_1 + Y_2$ .
- b The optimal consumption in period 2 is higher than in period 1.
- c If  $Y_1 > Y_2$ , then the consumer always saves in period 1.
- d If  $Y_2 > Y_1$ , then the consumer always borrows in period 1.
- e If the interest rate is zero, then the consumer's consumption is equal in both periods.

1.4 In order to ensure that more people are able to afford bread and in order to restrain bakers' greed, the government of Someplace introduces a price cap  $\bar{p}$  in the market for bread (which otherwise is completely unregulated). I.e. the price at which the goods are exchanged must not exceed  $\bar{p}$ . Assume that all agents in the market act as price takers.

- a Assume (only in this subquestion) that the price cap is strictly above the price that would obtain in the competitive equilibrium without any regulation. Statement: The consumer surplus with the price cap is strictly smaller than in the market without any regulation.
- b Assume now (only in this subquestion) that the price cap is strictly below the price that would obtain in the competitive equilibrium without any regulation. Statement: The consumer surplus with the price cap is at least as high as in the market without any regulation.
- c The introduction of a price cap yields a strict welfare loss compared to a market without any regulation.
- d The producer surplus with a price cap is weakly smaller than in a market without any regulation.
- e A smart choice of the price cap may yield a strict Pareto-improvement compared to an equilibrium allocation in a market without any regulation (i.e. at least one agent is strictly better off, without making the other agents strictly worse off).

1.5 Consider an exchange economy with two agents and two goods. There is a total of  $e_1$  units of good 1 and  $e_2$  units of good 2 that the agents own initially. The agents behave competitively. Assume that at least one general equilibrium exists for each distribution of the total endowments  $e_1$  and  $e_2$  among the agents.

- a An allocation that maximizes the sum of both agents' utilities is Pareto optimal.

- b** All points on the contract curve are Pareto-efficient allocations.
- c** There may be several different equilibria, where the consumers purchase the same quantities.
- d** The equilibrium price ratio in general depends on how the endowments are distributed among the agents.
- e** For each allocation  $x$  that is not an element of the contract curve, there is at least one allocation on the contract curve that is a Pareto-improvement compared to  $x$ .

## 2 Text Problems

**2.1** A firm sells insurance to students against the theft of bicycles. The firm has the options to either offer full insurance or no insurance at all to students. Before signing a contract with the firm, students may privately purchase locks in order to protect their property. The safety of the locks depends on their price as follows.

1. Price €0 (no lock): the probability of theft is equal to 50%.
2. Price €30: the probability of theft is equal to 30%.
3. Price €50: the probability of theft is equal to 20%.
4. Price €100: the probability of theft is equal to 10%.

The bicycle's value is  $A = €1.050$ . In case of theft the lock is destroyed and its value is €0.

A representative student's preferences are given by the money utility function

$$u = \sqrt{Y + A}.$$

In addition to the value of the bicycle, each student has a wealth of €300. ( $Y$  therefore is this wealth minus the price of the lock.) The students maximize their expected utility. The firm's costs are €0 per insurance policy.

**2.1.1 (N)** Determine the representative student's optimal expenditure on the lock if no insurance is offered.

**2.1.2 (N)** Determine the actuarially fair premium for full insurance if each student purchases the lock for 50 Euro and purchases full insurance. State the premium rounded to integer numbers.

2.1.3 (N) Determine a representative student's optimal expenditure for a lock if the premium is 212 Euro and the student purchases full insurance.

2.1.4 (N) Determine the insurance company's expected loss per student if the premium is 212 Euro, each student purchases full insurance and buys the lock from part 2.1.3. State the value rounded to integer numbers.

2.1.5 (N) Determine the actuarially fair premium for full insurance if the students do not purchase locks. State the premium rounded to integer numbers.

**2.2.** Britney may buy apples and pears. Apples and pears are not divisible. Let  $x_A$  be the quantity of apples and let  $x_P$  be the quantity of pears that Britney consumes. The price for apples is denoted by  $p_A$  and the price for pears is  $p_P$ . Britney's preferences are as follows. She weakly prefers the bundle  $x' = (x'_A, x'_P)$  to the bundle  $x'' = (x''_A, x''_P)$ , if  $x'_A + x'_A x'_P + x'_P \geq x''_A + x''_A x''_P + x''_P$ .

2.2.1 (N). Suppose that Britney is just able to afford each of the bundles  $(x_A^*, x_P^*) = (3, 0)$  and  $(x_A^{**}, x_P^{**}) = (0, 3)$ . I.e. if Britney purchases the bundle  $(x_A^*, x_P^*)$  or  $(x_A^{**}, x_P^{**})$ , then she has no money left. Determine the price ratio  $p_A/p_P$  in the market and state it rounded to integer numbers.

2.2.2 (N). Suppose that  $p_A = 1$ ,  $p_P = 2$ . What is Britney's income  $Y$  if she is just able to afford the bundle  $(x_A^{**}, x_P^{**}) = (0, 3)$ ?

2.2.3 (MC) Suppose that Britney's income is  $Y = 9$  and the prices are  $p_A = 3$  and  $p_P = 3$ . Exactly one of the following statements is true.

- a** There is exactly one optimal consumption bundle.
- b** At the optimum one necessarily has  $|\text{Marginal rate of substitution}| = |\text{Slope of the budget line}|$ .
- c** At the optimum one necessarily has  $|\text{Marginal rate of substitution}| < |\text{Slope of the budget line}|$ .
- d** Optimally Britney spends her entire income.
- e** None of the statements (a)-(d) is correct.

2.2.4 (N) Suppose  $p_A = 3$ ,  $p_P = 3$  and  $Y = 14$ . How much money does Britney not spend at the optimum?

2.2.5 (N) Suppose the same prices as in part 2.2.4, but the income is  $Y = 6$ . Determine the number of optimal consumption bundles.

**2.3** The preferences of two persons,  $A$  und  $B$ , are given by:

$$u^A(x_1^A, x_2^A) = \min(x_1^A, x_2^A) \quad \text{and} \quad u^B(x_1^B, x_2^B) = \ln x_1^B + \ln x_2^B$$

where  $\ln$  denotes the natural logarithm,  $u^j$  is the utility of agent  $j$  and  $x_i^j$  is the consumption of good  $i$  by agent  $j$ . The agents' endowments are given by

$$\begin{aligned} e_1^A &= 7, & e_2^A &= 9 \\ e_1^B &= 12, & e_2^B &= 10 \end{aligned}$$

The agents trade with each other as if they were on a competitive market, i.e. they consider the prices  $p_1$  and  $p_2$  as given. In the following calculations, make sure that intermediate results are accurate to 4 decimal places.

2.3.1 (MC) Exactly one of the following statements is true.

- a** The allocation  $(x_1^A, x_2^A, x_1^B, x_2^B) = (8, 10, 11, 10)$  is feasible.
- b** The allocation  $(x_1^A, x_2^A, x_1^B, x_2^B) = (5, 5, 7, 7)$  is an element of the Edgeworth Box.
- c** The contract curve is a straight line.
- d** The above endowments lie on the contract curve.
- e** None of the statements (a)-(d) is correct.

2.3.2 (N) Determine agent  $A$ 's equilibrium consumption of good 1 and state this quantity rounded to integer numbers.

2.3.3 (N) How many units of good 1 are sold by agent  $B$  in equilibrium? State this quantity rounded to integer numbers.

2.3.4 (N) Determine agent  $B$ 's equilibrium utility and state it rounded to integer numbers.

2.3.5 (N) Agent  $A$  destroys two units of good 2 and agent  $B$  destroys two units of good 1 before the agents interact in the market. Determine  $A$ 's consumption of good 1 in general equilibrium. State this quantity rounded to integer numbers.