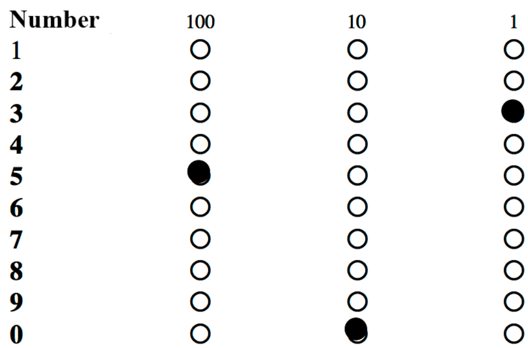
Version: A

Examination in Microeconomics A
Spring Term 2014 (1st Exam)

Handling of the exam

- Please check carefully whether your exam sheets are complete and correct, objections after the exam cannot be considered.
  - There are 2 **versions** of this exam, which are denoted by A and C respectively. Please check carefully, whether the version on the question sheet corresponds to the one on the solution sheet.
  - The **question sheet** (including the pages with the general remarks) consists of 7 pages. In addition there is a **solution sheet**, which consists of 3 pages.
- The use of resources other than a non-programmable calculator and at most one dictionary is not allowed. The use of other resources (e.g. programmable calculators, your own concept paper) leads to the disqualification from the exam.
- You have 120 minutes to solve the exam.
- The **exam** consists of 4 True/False problems and 3 Text Problems.
- For each **True/False Problem** and each of the 5 numerated statements that are made in the problem, you have to decide whether the statement is true (T) or false (F). Please mark T on the solution sheet if the statement is true for all the cases captured by the statement, and mark F for false otherwise. You will be awarded points according to the following rule: If your answer is correct, you obtain *1 point* per statement, and *0 points* otherwise. For each True/False Problem you can therefore obtain up to *5 points*. A priori, any subset of the 5 statements can be true.
- Each **Text Problem** has, on the one hand, a subproblem (MC) consisting of 5 statements denoted by a to e, each of which can be true (T) or false (F). Please mark T on the solution sheet if the statement is true for all the cases captured by the statement, and mark F for false otherwise. Points are awarded according to the same rule as for the True-/False problems described above. A priori, any subset of the 5 statements can be true.

  On the other hand, there are numerical subproblems (N), where you have to fill in the result on the solution sheet in encoded form. For each numerical subproblem you get *2 points* if answered correctly and *0 points* otherwise. For each Text Problem you can therefore obtain up to *13 points*. Here is an example on how to encode integers in the numerical subproblem: Suppose the solution to the question is **503**. Then this number has to be filled in as in the following figure:



Important: Mark the zero in the first column if the solution is a two-digit number. Similarly, mark the zero in the first and in the second column if the solution is a single-digit number.

- In total you can obtain up to *59 points*.

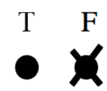
- You will pass the exam with certainty if you obtain at least *29 points* or if you are among the 75% best participants of the exam.

Handling of the solution sheet

- You **only** have to hand in the solution sheet at the end of the exam. Answers on concept sheets or on the question sheet will not be considered. We recommend that you fill in the solutions at the **end of the exam** in order to avoid corrections. Please start to fill in your answers **at least 5 minutes before the end of the exam**. The supervisors have orders to collect the solution sheets, even if you have not yet filled in your answers.

- *Please fill in the whole circle, do not mark answers with a cross!* Only *unambiguously legible* solutions can yield points. Please do not use TippEx to correct your answers! In the case that a circle was already filled in, but then you want to give no answer, first fill in another circle and then cross out both circles (see example). Please use dark colors (black or blue) and no pencil.

- Example: The answer is supposed to be T, answer F was filled in. Then in the end the solution sheet has to look like this:



- You must **sign** your **solution sheet** at the bottom.

Concerning the content of the exam

- If necessary, state your result rounded to the nearest integer.

Good Luck!

1 True-/ False Problems

1.1 Let X denote the set of bundles of two goods, each of which can be consumed in arbitrary non-negative quantities (i.e. sets of $\mathbf{R}_+$). Suppose that Lisa's preferences over X are described by a complete and transitive relation $\succeq$. Denote by $\succ$ the corresponding strict preference relation.

1. If $(3,3) \succeq (1,1)$ and Lisa's preferences are monotonic, then $(2,2) \succeq (1,1)$.

2. If $(9,5) \succeq (1,1)$ and Lisa's preferences are convex, then $(3,2) \succeq (1,1)$.

3. If $\succeq$ is continuous and monotonic, then there exists a utility function $u : X \to \mathbf{R}$ that represents Lisa's preferences.

4. If $(4,4) \succeq (10,0)$, $(4,4) \succeq (0,6)$, and $(5,3) \succeq (4,4)$, then Lisa's preferences cannot be convex.

5. For all $x, y, z, w \in X$ it is true that: if $x \succeq y$, $z \succeq w$ and $y \succ z$ then $x \succ w$.

1.2 Let X denote the set of bundles of two goods, each of which can be consumed in arbitrary non-negative quantities (i.e. sets of $\mathbf{R}_+$). Suppose that Lisa's preferences over X are monotonic and convex, and all of her indifference curves can be represented by differentiable functions. The prices of goods 1 and 2 are $p_1 = 3$ and $p_2 = 1$, respectively. Lisa has a budget of $m > 0$ to spend on goods 1 and 2. All numerated statements in this problem refer to an arbitrary consumption bundle (x_1^*, x_2^*) that is optimal for Lisa. Denote by $|MRS_{1,2}(x_1^*, x_2^*)|$ the absolute value of the marginal rate of substitution of good 2 for good 1 at the point (x_1^*, x_2^*).

1. If $x_1^* > 0$ and $x_2^* > 0$, then $|MRS_{1,2}(x_1^*, x_2^*)| = 3$.

2. If $x_1^* > 0$ and $x_2^* > 0$, then $|MRS_{1,2}(x_1^*, x_2^*)| = 1/3$.

3. If $x_1^* > 0$ and $x_2^* = 0$, then $|MRS_{1,2}(x_1^*, x_2^*)| \geq 3$.

4. If $x_1^* > 0$ and $x_2^* = 0$, then $|MRS_{1,2}(x_1^*, x_2^*)| \leq 1$.

5. If $x_1^* = 0$ and $x_2^* > 0$, then $|MRS_{1,2}(x_1^*, x_2^*)| \leq 3$.

1.3 Consider a firm with production function $f(x_1, x_2)$, where both inputs can be chosen in arbitrary non-negative quantities (i.e. sets of $\mathbf{R}_+$). Assume $f(x_1, x_2) > 0$ for all $x_1 > 0, x_2 > 0$, and $f(x_1, x_2) = 0$ otherwise. Moreover, assume that f is strictly increasing and continuous. Suppose that x_1 can be varied in the short run, but x_2 can only be varied in the long run and in the short run its level is fixed at $\bar{x}_2 > 0$. The unit price of input i is $p_i > 0$ for $i = 1, 2$. Suppose that the marginal product of input 1 is strictly decreasing for all x_1 if $x_2 = \bar{x}_2$.

1. The costs of the short-run production decision include setup costs equal to $p_2\bar{x}_2$.

2. The short run cost function includes fixed costs equal to $p_2\bar{x}_2$.

3. With respect to the short-run production decision, the costs of using input 2, $p_2\bar{x}_2$, are sunk costs.

4. The short run marginal cost function ($SRMC$) is strictly increasing.

5. The long run cost function includes setup costs equal to $p_2\bar{x}_2$.

1.4 Consider a competitive market for a single good (partial equilibrium analysis). The market demand for the good is $D(p) = p^{\varepsilon} > 0$ for all prices $p > 0$, where $\varepsilon < 0$ is a given parameter. The market supply is $S(p) = p^{\eta} > 0$ for all prices $p > 0$, where $\eta > 0$ is a given parameter. Define $p^* > 0$ such that $D(p^*) = S(p^*)$. Suppose that a unit tax $t = 2$ is introduced in this market. In the resulting competitive equilibrium, denote by p_D^* the price paid by the consumers and by p_S^* the price received by the firms.

1. $p_D^* - 2 = p_S^*$.

2. $D(p_S^* + 2) = S(p_S^*)$.

3. The price elasticity of the market supply equals η at all prices $p > 0$.

4. If $\eta > -\varepsilon$ then $p_D^* - 1 = p_S^* + 1$.

5. If $\eta = -\varepsilon$ then $p_D^* = p^* + 1$.

2 Text Problems

2.1 Mike has an initial wealth of $W > 0$. He can invest any amount A ($0 \leq A \leq W$) into a risky asset. With probability 1/2, he loses half of his investment (that is, in this event the investment returns $A/2$); also with probability 1/2, the risky investment returns $(1 + r)A$, where $r > 0$ is a given parameter. Mike invests the rest of his wealth, $W - A$, into a different asset that returns $W - A$ for sure. Mike's utility from any amount of money $X \geq 0$ is $U(X) = \sqrt{X}$. Mike is an expected utility maximizer.

2.1.1 (N) Determine Mike's amount of investment A if $r = 1/2$.

2.1.2 (N) Determine the smallest value of r such that Mike invests all of his money into the risky asset, that is, $A = W$.

2.1.3 (N) Find the value of r such that Mike invests two-thirds of his wealth into the risky asset, that is, $A = (2/3)W$. *Please enter* ***here the number 100r*** *into your solution sheet.*

2.1.4 (N) Determine the value of Mike's A if $r = 3/4$ and $W = 3$.

2.1.5 (MC) Which of the following statements is/are true/false?

a. Mike is risk-neutral.

b. Mike is risk-averse.

c. In the cases with $r < 1/2$, Mike would be better off if he could short-sell the risky asset, that is, if he could choose $A < 0$.

d. In the cases $1 \geq r > 1/2$, Mike would be better off if he could short-sell the sure asset, that is, if he could choose $A > W$.

e. In the case $r = 3$, Mike would be better off if he could short-sell the sure asset.

2.2 Consider a firm with a technology that is described by the production function $f(x_1, x_2) = (x_1)^{1/4}(x_2)^{1/4}$, where x_1 and x_2 are arbitrary non-negative input quantities (i.e. sets of $\mathbf{R}_+$). Suppose that the prices of the input goods are $p_1 = p_2 = 2$. For problems 2.2.1 to 2.2.4, assume that the firm produces 3 units of output with minimal costs.

2.2.1 (N) Suppose that, in the short run, the level of input 1 can be freely chosen while the level of input 2 is fixed at $\overline{x}_2 = 81$. Determine the cost of the firm's short-run decision.

2.2.2 (N) Determine the firm's choice of x_1 in the long run.

2.2.3 (N) Determine the firm's choice of x_2 in the long run.

2.2.4 (N) Determine the firm's long run production cost.

2.2.5 (MC) Which of the following statements is/are true/false?

a. The firm's technology has increasing returns to scale.

b. The firm's technology has decreasing returns to scale.

c. The firm's long run average cost function is strictly increasing in the output quantity.

d. The firm's long run average cost function is strictly decreasing in the output quantity.

e. The firm's long run average cost function is constant in the output quantity.

2.3 Consider an exchange economy with two goods, 1 and 2, that may be consumed in arbitrary non-negative quantities (i.e. sets of $\mathbf{R}_+$). There are 2 equal sized groups of consumers that we call type-A consumers and type-B-consumers. Each consumer is a price-taker. The preferences of each type-A consumer are represented by the utility function $u^A(x_1^A, x_2^A) = x_1^A x_2^A + 8$, where x_i^A denotes the quantity of good $i = 1, 2$ that she consumes. Each type-A consumer has the same initial endowment denoted by $e^A = (e_1^A, e_2^A)$, with $e_1^A > 0$ and $e_2^A > 0$. The preferences of each type-B consumer are represented by the utility function $u^B(x_1^B, x_2^B) = 9x_1^B + 9x_2^B$, where x_i^B denotes the quantity of good $i = 1, 2$ that she consumes. Each type-B consumer has the same initial endowment denoted by $e^B = (e_1^B, e_2^B)$, with $e_1^B > 0$ and $e_2^B > 0$. A competitive equilibrium is described by the prices p_1^*, p_2^* of good 1 and good 2, respectively, the bundle consumed by each type-A consumer, (x_1^{A*}, x_2^{A*}), and the bundle consumed by each type-B consumer, (x_1^{B*}, x_2^{B*}).

For problems 2.3.1 to 2.3.4, assume that $e^A = (2, 4)$ and $e^B = (10, 4)$.

2.3.1 (N) Calculate x_1^{A*}.

2.3.2 (N) Calculate x_2^{A*}.

2.3.3 (N) Calculate x_1^{B*}.

2.3.4 (N) Determine p_1^* if $p_2^* = 5$.

2.3.5 (MC) Which of the following statements is/are true/false?

a. If $e_1^A + e_1^B > e_2^A + e_2^B$ and $e_2^A > e_1^B$, then there exists a competitive equilibrium in which $x_2^{B*} = 0$.

b. If $e_1^A + e_1^B = e_2^A + e_2^B$, then $p_1^* = p_2^*$ in any competitive equilibrium.

c. If $e_1^A \neq e_2^A$ and $e_1^A + e_1^B = e_2^A + e_2^B$, then the competitive equilibrium is such that each type-A consumer is strictly better off than with her initial endowment.

d. If $e_1^A + e_1^B < e_2^A + e_2^B$, then there exist Pareto efficient points at which the type-B consumers consume nothing of good 1.

e. For arbitrary endowments e^A and e^B, every point on the contract curve is Pareto efficient.