Tail Risk Hedging and Regime Switching
Online Appendix

August 2016

B Tail Risk Hedging for Simple Elliptical Distributions

In this appendix, we analyze the case of simple elliptical distributions, which we criticized in the introduction and then briefly summarized at the end of Section 3.1. In particular, we discuss conditions under which minimum-variance hedging will be a good approximation to tail-risk-minimal hedging or will even be equivalent. Therefore, we review the results of Theorem 1 for $K = 1$. This corresponds to

$$\begin{pmatrix} \boldsymbol{R}_S \\ \boldsymbol{R}_F \end{pmatrix} \sim \mathcal{E}_{N+M}\left(\begin{pmatrix} \boldsymbol{\mu}_S \\ \boldsymbol{\mu}_F \end{pmatrix}, \begin{pmatrix} \boldsymbol{\Sigma}_S & \boldsymbol{\Sigma}_{FS} \\ \boldsymbol{\Sigma}_{SF}' & \boldsymbol{\Sigma}_F \end{pmatrix}, g\right) \tag{B.1}$$

and for VaR_α and CVaR_α as a function of the hedging weights it implies that

$$v_\alpha(\boldsymbol{h}) = \mu_L(\boldsymbol{h}) + \sigma_L(\boldsymbol{h})\ z_{1-\alpha}(g) \quad \text{and} \quad c_\alpha(\boldsymbol{h}) = \mu_L(\boldsymbol{h}) + \sigma_L(\boldsymbol{h})\ \lambda_{1-\alpha}(g), \tag{B.2}$$

where $\mu_L(\boldsymbol{h}) = -\boldsymbol{w}' \cdot \boldsymbol{\mu}_S + \boldsymbol{h}' \cdot \boldsymbol{\mu}_F$ and $\sigma_L(\boldsymbol{h}) = \boldsymbol{w}' \cdot \boldsymbol{\Sigma}_S \cdot \boldsymbol{w} - 2\ \boldsymbol{w}' \cdot \boldsymbol{\Sigma}_{SF} \cdot \boldsymbol{h} + \boldsymbol{h}' \cdot \boldsymbol{\Sigma}_F \cdot \boldsymbol{h}$. The generator specific constants $z_{1-\alpha}$ and $\lambda_{1-\alpha}$ can be derived from the distribution of $Z \sim \mathcal{E}_1(0, 1, g)$ as $(1-\alpha)$-quantile $z_{1-\alpha}(g) = q_{1-\alpha}[Z]$ and a corresponding tail conditional expectation $\lambda_{1-\alpha}(g_1) = \mathbb{E}[Z \mid Z \geq z_{1-\alpha}(g)]$. Note that in contrast to $z_k(\boldsymbol{h})$ and $\lambda_k(\boldsymbol{h})$ in the mixture case, $z_{1-\alpha}(g)$ and $\lambda_{1-\alpha}(g)$ do not depend on the hedging strategy but only on the generator of the distribution and the chosen confidence level $1-\alpha$.[1] Furthermore, we

[1] For a normal or standardized t-distribution, the expressions provided in Section 3.2 can be used here. See Landsman and Valdez (2003) for a comprehensive discussion of elliptical risk models and the

obtain the following simplified FOCs from Theorem 1.[2]

Corollary 1 *Under (R1) and (M1) with $K = 1$ the* VaR_α*-minimal hedge vector* $\boldsymbol{h}^*_{\text{VaR}}$ *and the* CVaR_α*-minimal hedge vector* $\boldsymbol{h}^*_{\text{CVaR}}$ *solve*

$$\boldsymbol{\mu}_F + \frac{\boldsymbol{\Sigma}_{FL}(\boldsymbol{h}^*_{\text{VaR}})}{\sigma_L(\boldsymbol{h}^*_{\text{VaR}})}\; z_{1-\alpha}(g) = \mathbf{0}_M \quad \text{and} \quad \boldsymbol{\mu}_F + \frac{\boldsymbol{\Sigma}_{FL}(\boldsymbol{h}^*_{\text{CVaR}})}{\sigma_L(\boldsymbol{h}^*_{\text{CVaR}})}\; \lambda_{1-\alpha}(g) = \mathbf{0}_M, \tag{B.3}$$

where $\boldsymbol{\Sigma}_{FL}(\boldsymbol{h}) = -\boldsymbol{\Sigma}'_{SF} \cdot \boldsymbol{w} + \boldsymbol{\Sigma}_F \cdot \boldsymbol{h}$.

In this setting, not only CVaR_α- but also VaR_α-minimal hedging is a convex optimization problem for $\alpha < 0.5$.[3] Moreover, by subtracting $\boldsymbol{\mu}_F$ in (B.3) and solving for $\boldsymbol{h}^*_{\text{MVaR}}$ and $\boldsymbol{h}^*_{\text{MCVaR}}$, it follows that

$$\boldsymbol{h}^*_{\text{MVaR}} = \boldsymbol{h}^*_{\text{MCVaR}} = \boldsymbol{h}^*_{\text{var}} = \boldsymbol{\Sigma}_F^{-1} \cdot \boldsymbol{\Sigma}'_{SF} \cdot \boldsymbol{w}. \tag{B.4}$$

Thus, in this case, minimum-variance and tail-risk-minimal hedging policies coincide, which is similar to the equivalence between variance and VaR_α-based portfolio optimization problems if the mean is fixed (Embrechts et al., 2002, Theorem 1).[4] Obviously, the same applies to VaR_α- and CVaR_α-minimal hedging policies if $\boldsymbol{\mu}_F = \mathbf{0}_M$. Summing up, *using elliptical distributions*, differences between minimum-variance and tail-risk-minimal hedges are only due to the contributions of the hedging positions to the expected portfolio return.

To improve our understanding of the case $\boldsymbol{\mu}_F \neq \mathbf{0}_M$, we finally analyze $N = M = K = 1$, which corresponds to an 1:1 hedging problem under the assumption of an ordinary elliptical

derivation of the relevant constants in general.

2 Another important distinction to the mixture case is that these results could also be derived by immediately differentiating the risk measures in (B.2) with respect to $\boldsymbol{h}$.

3 This follows from the convexity/subadditivity of VaR_α for elliptical distributions (Embrechts et al., 2002, Theorem 1).

4 This result can also be seen from the form of the risk measures in (B.2) because from those we obtain $\text{MVaR}_\alpha[L_H(\boldsymbol{h})] = \sigma_L(\boldsymbol{h})\; z_{1-\alpha}(g)$ and $\text{MCVaR}_\alpha[L_H(\boldsymbol{h})] = \sigma_L(\boldsymbol{h})\; \lambda_{1-\alpha}(g)$.

distribution for the spot and the futures return.[5] The resulting simplified version of (M1) can be written as

$$\begin{pmatrix} R_S \\ R_F \end{pmatrix} \sim \mathcal{E}_2(\begin{pmatrix} \mu_S \\ \mu_F \end{pmatrix}, \begin{pmatrix} \sigma_S^2 & \rho\sigma_S\sigma_F \\ \rho\sigma_S\sigma_F & \sigma_F^2 \end{pmatrix}, g). \tag{B.5}$$

Under the additional restrictions

$$|z_{1-\alpha}|\ \sigma_F > |\mu_F| \qquad \text{or} \qquad |\lambda_{1-\alpha}|\ \sigma_F > |\mu_F|, \tag{B.6}$$

we can solve (B.3) for the hedging weights, which are [6]

$$h^*_{\mathrm{VaR}} = \frac{\sigma_S}{\sigma_F}\left(\rho - \mu_F\sqrt{\frac{1-\rho^2}{z^2_{1-\alpha}(g)\sigma_F^2 - \mu_F^2}}\right), \tag{B.7}$$

$$h^*_{\mathrm{CVaR}} = \frac{\sigma_S}{\sigma_F}\left(\rho - \mu_F\sqrt{\frac{1-\rho^2}{\lambda^2_{1-\alpha}(g)\sigma_F^2 - \mu_F^2}}\right). \tag{B.8}$$

(B.7) was proved by Hung et al. (2006) for the Gaussian case and generalized in Albrecht (2011) for elliptical distributions. These expressions allow for an instructive analytic comparison of minimum-variance and tail-risk-minimal hedging, which complements the discussion in Alexander and Baptista (2002, 2004) on VaR_α- and CVaR_α-based portfolio selection problems.

We see again the correspondence of variance and quantile-based hedging for $\mu_F = 0$. In this case, the conditions for the existence of a tail-risk-minimizing strategy will always be satisfied. In general, theses conditions define a lower and upper limit for the expected futures returns. If the absolute value of the expected return of the futures contract is higher than the standard deviation scaled by $|z_{1-\alpha}|$ or $|\lambda_{1-\alpha}|$, VaR_α and CVaR_α can be made arbitrarily small (negative) by increasing or decreasing h. Note that usually (B.6)

[5] The assumption $N = 1$ is not crucial here. For any given multivariate $\boldsymbol{R}_S$ and the corresponding weight vector $\boldsymbol{w}$, we can deduce the distribution of $\boldsymbol{w}' \cdot \boldsymbol{R}_S$ and its correlation with $\boldsymbol{R}_F$.

[6] Note that the calculations involve solving a quadratic equation. Nevertheless, the hedge ratios are unique because only one solution of the quadratic equation solves the original problems in (B.3).

will be satisfied because for small values of α, $z_{1-\alpha}$ and $\lambda_{1-\alpha}$ are greater than one and at typical investment or hedging horizons, the magnitude of σ_F dominates that of μ_F.

The differences between variance and tail-risk-minimal hedging strategies follow simply from (B.7) and (B.8). In the case of VaR_α, e.g., we obtain

$$h^*_{\mathrm{VaR}} - h^*_{\mathrm{var}} = -\mu_F \sqrt{\frac{1-\rho^2}{z^2_{1-\alpha}\sigma^2_F - \mu^2_F}}, \tag{B.9}$$

which leads to the following observations: (i) The variation increases in the absolute level of the mean of the futures return. If the mean of the hedging instrument is positive, the hedging amount is reduced compared to the minimum-variance position. This can be attributed to a positive contribution of the futures position to the expected loss of the hedged portfolio. (ii) Both the variance of the futures return and an increasing confidence level lower the difference. (iii) For the same confidence level, the difference between minimum VaR_α and minimum-variance hedging is greater than the difference between minimum CVaR_α and minimum-variance hedging. (iv) Eventually, we see that the difference is only relevant if there is some basis risk because it decreases in the level of correlation.[7] For a typical parameter constellation such as $\mu_F = 0.01, \sigma_F = 0.05, \sigma_S = 0.05, \rho = 0.90, \alpha = 0.01$, and assuming joint normality of the returns, we obtain $h^*_{\mathrm{var}} = 0.90$ and $h^*_{\mathrm{VaR}} = 0.86$, so that the minimum-variance approach can be seen as a good approximation to tail-risk-minimal hedging.

We now provide closed form expressions for the attainable reductions on the level of VaR_α and CVaR_α derived from (B.2). We first calculate the VaR_α for the optimal strategy using

[7] The first three effects (i) - (iii) are not surprising given (B.2), which shows that minimizing VaR_α and CVaR_α under elliptical distributions corresponds to a mean-standard deviation optimization, where the trade-off between these two objectives is described by the quantile risk constants $z_{1-\alpha}$ and $\lambda_{1-\alpha}$.

$h^*_{\mathrm{VaR}_\alpha}$ from (B.7). Note that

$$\mu_L(h^*_{\mathrm{VaR}}) = -\mu_S + \mu_F\ \frac{\sigma_S}{\sigma_F}\ \rho - \mu_F^2\ \frac{\sigma_S}{\sigma_F}\ \sqrt{\frac{1-\rho^2}{z^2\sigma_F^2 - \mu_F^2}}, \tag{B.10}$$

$$\sigma_L(h^*_{\mathrm{VaR}_\alpha}) = \sigma_S \cdot \sigma_F \cdot z \cdot \sqrt{\frac{1-\rho^2}{z^2\sigma_F^2 - \mu_F^2}} \tag{B.11}$$

and thus

$$\mathrm{VaR}_\alpha(h^*_{\mathrm{VaR}}) = -\ \mu_S + \mu_F \cdot \rho \cdot \frac{\sigma_S}{\sigma_F} + \frac{\sigma_S}{\sigma_F}\ \sqrt{1-\rho^2}\ \sqrt{z^2\sigma_F^2 - \mu_F^2}. \tag{B.12}$$

Using the minimum-variance strategy, we obtain

$$\mu_L(h^*_{\mathrm{var}}) = -\mu_S + \rho \cdot \frac{\sigma_S}{\sigma_F} \cdot \mu_F, \tag{B.13}$$

$$\sigma_L^2(h^*_{\mathrm{var}}) = \sigma_S^2(1-\rho^2). \tag{B.14}$$

The corresponding VaR_α is

$$\mathrm{VaR}_\alpha(h^*_{\mathrm{var}}) = -\mu_S + \mu_F \cdot \rho \cdot \frac{\sigma_S}{\sigma_F} + z_{1-\alpha}(g) \cdot \sigma_S \cdot \sqrt{1-\rho^2}. \tag{B.15}$$

In the given example with $\mu_S = 0.01$, we obtain $\mathrm{VaR}_\alpha(h^*_{\mathrm{VaR}}) = 0.0495$ and $\mathrm{VaR}_\alpha(h^*_{\mathrm{var}}) = 0.0497$, showing that also on the level of the risk measures, the magnitude of reductions is very small. In general, it holds that

$$\mathrm{VaR}_\alpha(h^*_{\mathrm{var}}) - \mathrm{VaR}_\alpha(h^*_{\mathrm{VaR}}) = \sigma_S\ \sqrt{1-\rho^2}\left[z_{1-\alpha}(g) - \sqrt{z^2_{1-\alpha}(g) - \frac{\mu_F^2}{\sigma_F^2}}\right]. \tag{B.16}$$

The same reasoning can be used for CVaR_α-minimal hedging, which yields

$$\mathrm{CVaR}_\alpha(h^*_{\mathrm{var}}) - \mathrm{CVaR}_\alpha(h^*_{\mathrm{CVaR}}) = \sigma_S \ \sqrt{1-\rho^2}\left[\lambda_{1-\alpha}(g) - \sqrt{\lambda^2_{1-\alpha}(g) - \frac{\mu_F^2}{\sigma_F^2}}\right]. \qquad \text{(B.17)}$$

Applying $\left|\sqrt{a}-\sqrt{b}\right| \leq \sqrt{|a-b|}$ to (B.16), we obtain the following upper bound for the VaR_α reduction

$$\mathrm{VaR}_\alpha(h^*_{\mathrm{var}}) - \mathrm{VaR}_\alpha(h^*_{\mathrm{VaR}}) \leq |\mu_F| \ \frac{\sigma_S}{\sigma_F} \ \sqrt{1-\rho^2}, \qquad \text{(B.18)}$$

which confirms the importance of $|\mu_F|$ and ρ for the relevance of tail-risk-minimal hedging in the elliptical case. Since this reasoning is independent of the involved (risk) constants, the same bound holds for CVaR_α hedging.

C Further Empirical Results

In this section, we provide further empirical results for our in-sample and out-of-sample analyses and especially for the robustness checks. In addition to estimation results, we show hedging results based on simple elliptical distributions and provide plots of smoothed probabilities for our RS models. Complementing Figure 3 in the main text, we provide exceedance correlations and lower tail dependence functions for (P1).

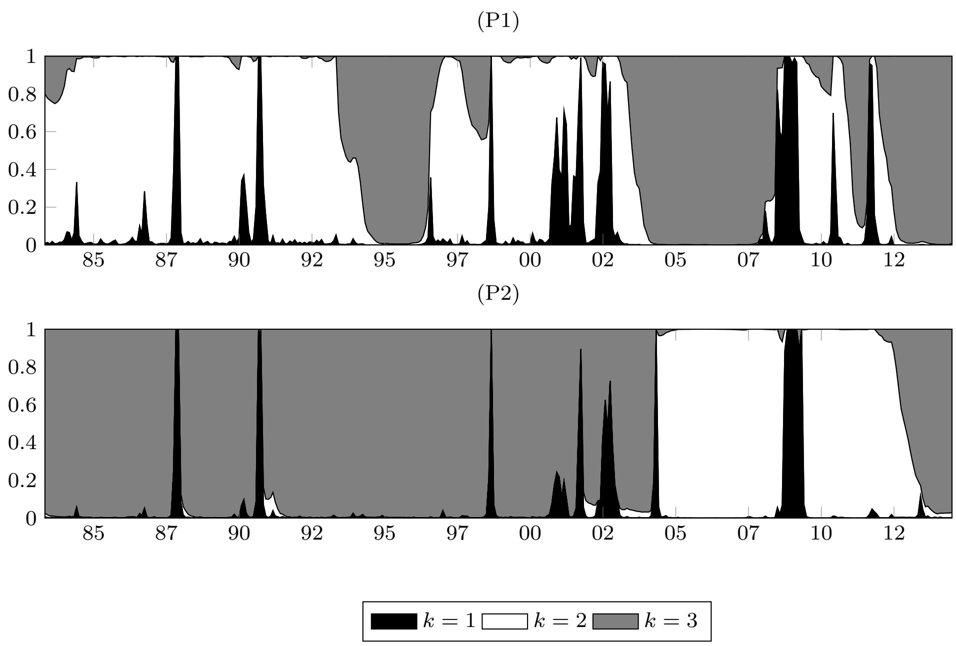

Figure C.1: Smoothed Probabilities for the Three-State RS Models Presented in Table 3

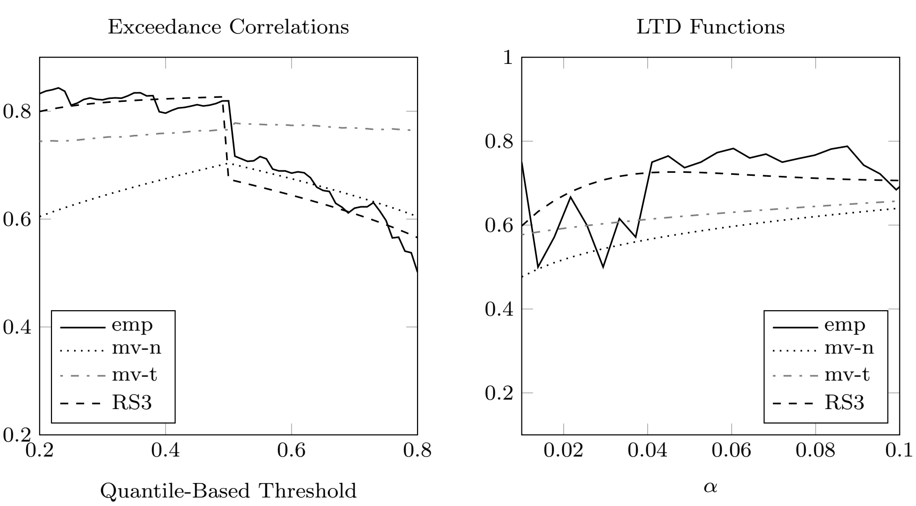

Figure C.2: Exceedance Correlations and Lower Tail Dependence Functions for the Returns of (P1) and the S&P futures

Table C.1: Parameter Estimates, Simple Elliptical Distributions

	(P1)		(P2)	
	par	s.e.	par	s.e.
Multivariate normal distribution				
μ_S	0.80	(0.15)	0.81	(0.17)
μ_F	0.50	(0.23)	0.50	(0.23)
σ_S	2.98	(0.11)	3.26	(0.12)
σ_F	4.42	(0.16)	4.42	(0.16)
ρ_{SF}	86.52	(1.30)	81.40	(1.74)
Multivariate standardized t-distribution				
ν	4.48	(0.78)	4.08	(0.64)
μ_S	1.08	(0.13)	1.14	(0.13)
μ_F	0.90	(0.20)	0.88	(0.19)
σ_S	2.94	(0.18)	3.07	(0.21)
σ_F	4.44	(0.28)	4.53	(0.31)
ρ_{SF}	85.41	(1.61)	79.74	(2.15)

In-sample parameter estimates for bivariate normal and standardized t-distributions.

Table C.2: Parameter Estimates, EM Algorithm

Panel A: RS $K=2$	(P1)		(P2)	
	par	s.e.	par	s.e.
State 1				
$\mu_{S,1}$	-0.57	(0.83)	-2.60	(1.54)
$\mu_{F,1}$	-1.61	(1.15)	-4.18	(1.71)
$\sigma_{S,1}$	4.89	(0.58)	6.59	(1.63)
$\sigma_{F,1}$	6.96	(0.67)	7.42	(1.19)
$\rho_{SF,1}$	87.29	(2.64)	83.10	(4.25)
State 2				
$\mu_{S,2}$	1.18	(0.14)	1.25	(0.17)
$\mu_{F,2}$	1.08	(0.20)	1.1	(0.26)
$\sigma_{S,2}$	2.03	(0.12)	2.17	(0.16)
$\sigma_{F,2}$	3.17	(0.17)	3.43	(0.25)
$\rho_{SF,2}$	83.52	(2.18)	76.33	(2.60)
Transition matrix				
q_{11}	83.3	(7.4)	63.3	(11.6)
q_{21}	4.6	(1.7)	4.7	(2.3)
Starting probabilities				
π_1	0.00	-	0.00	-
π_2	100.00	-	100.00	-

Panel B: RS $K=3$	(P1)		(P2)	
	par	s.e.	par	s.e.
State 1				
$\mu_{S,1}$	-3.70	(6.66)	-4.00	(2.55)
$\mu_{F,1}$	-6.17	(10.66)	-6.12	(2.61)
$\sigma_{S,1}$	4.85	(1.36)	7.85	(2.02)
$\sigma_{F,1}$	6.29	(4.42)	8.11	(1.55)
$\rho_{SF,1}$	82.69	(24.59)	82.25	(5.75)
State 2				
$\mu_{S,2}$	1.46	(0.34)	1.14	(0.17)
$\mu_{F,2}$	1.36	(0.44)	1.01	(0.30)
$\sigma_{S,2}$	2.48	(0.49)	2.08	(0.20)
$\sigma_{F,2}$	4.01	(0.50)	3.73	(0.30)
$\rho_{SF,2}$	81.13	(3.28)	74.00	(4.41)
State 3				
$\mu_{S,3}$	0.97	(0.21)	1.05	(0.36)
$\mu_{F,3}$	0.93	(0.30)	0.69	(0.48)
$\sigma_{S,3}$	1.79	(0.15)	3.13	(0.31)
$\sigma_{F,3}$	2.52	(0.24)	3.57	(0.38)
$\rho_{SF,3}$	88.32	(2.24)	92.09	(2.19)
Transition matrix				
q_{11}	62.1	(34.7)	61.3	(14.0)
q_{12}	37.9	(72.0)	27.6	(24.1)
q_{21}	4.7	(4.9)	2.9	(1.7)
q_{22}	92.8	(3.1)	97.1	(13.7)
q_{31}	2.8	(2.6)	1.4	(2.3)
q_{32}	0.5	(1.0)	1.1	(4.9)
Starting probabilities				
π_1	0.00	-	0.00	-
π_2	100.00	-	100.00	-
π_3	0.00	-	0.00	-

In-sample parameter estimates for the bivariate RS models with two and three normal components obtained with the unconstrained EM algorithm (Hamilton, 1990).

Table C.3: Parameter Estimates, Monte-Carlo Simulations

			skewed-t margins		Copula parameters	
	mean	std	ν_{st}	λ_{st}	ν	ρ
S_1	0.81	3.26	3.46	-0.22	6.08	80.68
F_1	0.50	4.42	5.69	-0.22		

This table contains parameter estimates of the copula model used in our Monte-Carlo experiment. The parameters of the skewed-t distribution are obtained from MLE. The copula parameters are obtained after transforming the data with their marginal empirical distribution functions.

Table C.4: Parameter Estimates, Composite Hedging

Panel A: bivariate model	par	s.e.		par	s.e.
State 1			*State 2*		
$\mu_{S,1}$	-10.34	(2.37)	$\mu_{S,2}$	0.78	(0.40)
$\mu_{F,1}$	-8.88	(1.99)	$\mu_{F,2}$	-0.20	(0.90)
$\sigma_{S,1}$	5.59	(1.87)	$\sigma_{S,2}$	3.33	(0.38)
$\sigma_{F,1}$	4.56	(1.48)	$\sigma_{F,2}$	5.92	(0.75)
$\rho_{SF,1}$	99.35	(0.55)	$\rho_{SF,2}$	67.69	(5.96)
State 3			*Transition matrix*		
$\mu_{S,3}$	1.07	(0.16)	q_{11}	29.2	(18.0)
$\mu_{F,3}$	1.13	(0.23)	q_{21}	2.6	(1.9)
$\sigma_{S,3}$	1.98	(0.13)	q_{31}	0.8	(0.9)
$\sigma_{F,3}$	2.92	(0.20)	q_{12}	54.3	(25.7)
$\rho_{SF,3}$	52.20	(5.99)	q_{22}	88.7	(7.3)
			q_{32}	3.8	(2.1)
Stationary distribution					
π_1	1.9				
π_2	31.6				
π_3	66.5				

Panel B: trivariate model	par	s.e.		par	s.e.
State 1			*State 2*		
$\mu_{S,1}$	-5.45	(6.36)	$\mu_{S,2}$	0.90	(2.22)
$\mu_{F_1,1}$	-3.74	(5.79)	$\mu_{F1,2}$	-0.11	(3.02)
$\mu_{F_2,1}$	-22.42	(6.39)	$\mu_{F2,2}$	2.89	(2.62)
$\sigma_{S,1}$	7.28	(2.31)	$\sigma_{S,2}$	3.34	(1.41)
$\sigma_{F_1,1}$	7.01	(2.08)	$\sigma_{F1,2}$	6.50	(1.74)
$\sigma_{F_2,1}$	12.41	(2.20)	$\sigma_{F2,2}$	13.57	(2.19)
$\rho_1[S, F_1]$	99.07	(0.54)	$\rho_2[S, F_1]$	69.97	(15.30)
$\rho_1[S, F_2]$	3.13	(22.82)	$\rho_2[S, F_2]$	47.87	(36.94)
$\rho_1[F_1, F_2]$	-7.47	(22.67)	$\rho_2[F_1, F_2]$	3.12	(70.73)
State 3			*Transition matrix*		
$\mu_{S,3}$	0.99	(0.48)	q_{11}	50.2	(15.5)
$\mu_{F1,3}$	0.87	(0.59)	q_{12}	36.6	(81.6)
$\mu_{F2,3}$	1.02	(0.67)	q_{21}	0.0	(48.4)
$\sigma_{S,3}$	2.21	(0.25)	q_{22}	83.3	(42.3)
$\sigma_{F1,3}$	3.25	(0.35)	q_{31}	2.2	(6.0)
$\sigma_{F2,3}$	7.24	(1.37)	q_{32}	3.0	(4.7)
$\rho_3[S, F_1]$	55.87	(5.00)			
$\rho_3[S, F_2]$	62.40	(6.43)			
$\rho_3[F_1, F_2]$	-4.83	(11.35)			
Stationary distribution					
π_1	3.4				
π_2	21.1				
π_3	75.5				

Panel A contains in-sample parameter estimates for a bivariate RS model with three normal components fitted to the joint distribution of (P3) and S&P futures returns. Panel B contains estimation results for a trivariate RS model with three normal components fitted to the the joint distribution of (P3), S&P futures and oil futures returns. Robust standard errors are reported.

Table C.5: Parameter Estimates, RST3

	(P1)		(P2)	
	par	s.e.	par	s.e.
ν	244.7	(26.5)	6.3	(1.4)
State 1				
$\mu_{S,1}$	-3.76	(5.22)	0.53	(0.66)
$\mu_{F,1}$	-6.27	(8.35)	0.26	(0.74)
$\sigma_{S,1}$	4.84	(1.18)	5.33	(0.75)
$\sigma_{F,1}$	6.24	(3.55)	5.72	(0.63)
$\rho_{SF,1}$	82.53	(20.16)	93.74	(1.46)
State 2				
$\mu_{S,2}$	1.46	(0.33)	1.43	(0.18)
$\mu_{F,2}$	1.37	(0.44)	1.23	(0.22)
$\sigma_{S,2}$	2.49	(0.41)	2.00	(0.16)
$\sigma_{F,2}$	4.02	(0.42)	2.47	(0.18)
$\rho_{SF,2}$	81.03	(3.31)	69.17	(5.35)
State 3				
$\mu_{S,3}$	0.98	(0.20)	0.90	(0.18)
$\mu_{F,3}$	0.93	(0.30)	0.60	(0.33)
$\sigma_{S,3}$	1.80	(0.15)	2.67	(0.20)
$\sigma_{F,3}$	2.54	(0.26)	4.94	(0.32)
$\rho_{SF,3}$	88.36	(2.22)	79.32	(3.00)
Transition matrix				
q_{11}	61.4	(30.9)	97.4	(9.3)
q_{12}	38.5	(59.7)	2.1	(2.1)
q_{21}	4.6	(4.2)	0.9	(0.7)
q_{22}	92.8	(3.0)	97.8	(1.4)
q_{31}	2.8	(2.6)	0.0	(0.7)
q_{32}	0.8	(1.3)	1.0	(0.5)
Stationary distribution				
π_1	9.2	-	12.7	-
π_2	53.2	-	35.0	-
π_3	37.6	-	52.3	-

In-sample parameter estimates for bivariate RS models with three t-distributed components fitted to the bivariate distribution of (P1) and (P2) and the S&P futures returns. Robust standard errors are reported.

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