Ambiguity and The Tradeoff Theory of Capital Structure

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Abstract

We examine the impact ambiguity, or Knightian uncertainty, has on the capital structure decision. A static tradeoff theory model is developed in which agents are both ambiguity and risk averse. The model generates the well known prediction that increased risk—the uncertainty over known possible outcomes—leads firms to decrease leverage. Conversely, the model predicts that greater ambiguity—the uncertainty over the probabilities associated with the outcomes—leads firms to increase leverage. Using a theoretically motivated measure of ambiguity, our empirical analysis presents evidence consistent with the prediction that ambiguity has an important and distinct impact on capital structure choice.

Keywords and Phrases: Capital Structure, Ambiguity aversion, Ambiguity measure.

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1 Introduction

Almost every financial decision entails ambiguity (Knightian uncertainty).[1] In an influential paper, Ellsberg (1961) demonstrates that individuals act as if they are averse to this aspect of uncertainty.[2] Nevertheless, the canonical results in finance have been developed using expected utility theory, a theory that can be interpreted as either assuming away ambiguity or assuming that individual preferences are neutral to ambiguity. If indeed ambiguity and aversion to it play a significant role in financial decision-making, ignoring this aspect of uncertainty and individual related preferences implies that our characterizations of optimizing behavior are at best incomplete and at worst misleading.

We introduce ambiguity and aversion to ambiguity into the static tradeoff theory of corporate capital structure in hopes of providing new insights into capital structure decisions. Examining leverage decisions through the lens of a financial market with ambiguity and risk averse investors may serve to further enhance our understanding of corporate decisions. The market's valuation of a firm's chosen mix of securities is the focus of the capital structure question, making the consequences of ambiguity a central concern. Furthermore, most empirical studies of leverage, including literature standards such as Titman and Wessels (1988), take no account of ambiguity or aversion to ambiguity. To test the predictions delivered by theoretical model, we adapt recent empirical models of the static tradeoff theory of corporate capital structure. Our empirical tests provide strong evidence that ambiguity is an important explanatory variable for leverage.

The famous theorem of Modigliani and Miller (1958) asserts that, with perfect capital markets, two firms generating the same distribution of future cash flow will have the same market value regardless of their financing choices. Standard tradeoff theory frictions have been used to establish an interior optimum (e.g., Kraus and Litzenberger, 1973) for the financing decision. We extend these theories to account for ambiguity and ambiguity aversion. We show that in a perfect capital market, Modigliani and Miller's irrelevance proposition is maintained under ambiguity aversion. This can be seen using the familiar argument that buying the debt and the equity of a levered firm results in the investor obtaining claims to cash flows that are equivalent to the equity cash flow of an unlevered firm. Stated in an alternative way, while the cost of debt and the cost of equity capital both increase with an increase in ambiguity, the weighted average cost of capital remains invariant to changes in leverage

[1] *Risk* refers to as a case in which the event to be realized is *a-priori* unknown, but the odds of all the possible events are perfectly known. *Ambiguity*, or *Knightian uncertainty*, refers to the case where not only is the event to be realized *a-priori* unknown, but also the odds of all possible events are either unknown or not uniquely assigned. Throughout the paper the term "*uncertainty*" is used to refer to the aggregation of risk and ambiguity.

[2] Ellsberg (1961) demonstrates that in the presence of ambiguity, individuals typically violate the independence ("Sure-Thing Principle") axiom of expected utility theory.

for a given level of ambiguity.

The increase in the cost of equity (a lower valuation for levered equity with a given face value of debt) that accompanies an increase in ambiguity can be used to highlight another standard concern in the capital structure literature: the over-investment problem. Agency problems have played a prominent role in the capital structure literature (e.g., Jensen and Meckling, 1976). For a given debt level, increasing the risk of the firm's cash flow increases the value of the levered equity following the usual intuition from option pricing. This suggests that decision-makers who maximize the value of levered equity will trade off the (net present) value of an investment opportunity against its risk. This tradeoff has been argued to motivate negative net present value investment in projects with sufficient levels of risk. With an increase in ambiguity the opposite occurs: an increase in ambiguity reduces the value of levered equity. Using the intuition from option pricing, increased ambiguity reduces the value of a call option, because the increase in ambiguity causes ambiguity averse investors to reduce the perceived probabilities of positive net payoffs (e.g., Izhakian and Yermack, 2017; Augustin and Izhakian, 2019). Net present value would, therefore, be sacrificed by a manager seeking to maximize the value of the existing equity only if the investment achieved a sufficient *reduction* in ambiguity. Alternatively, consistent with Garlappi et al. (2017), investments that entail an increase in ambiguity may suffer from an under-investment problem. Investments that increase both ambiguity and risk may or may not be attractive to managers seeking to maximize the value of a firm's levered equity. Ambiguity aversion may, therefore, mitigate over-investment incentives of managers acting in the interests of shareholders.

When taxes and bankruptcy costs are introduced into the model, the theory predicts that while an increase in the level of risk results in a reduction in the use of debt financing, an increase in the level of ambiguity is associated with an increase in the use of debt. The economic intuition explaining why higher risk is associated with less extensive use of leverage (e.g., Kraus and Litzenberger, 1973) is maintained in the model with ambiguity aversion. At the optimal leverage, the marginal cost of bankruptcy equals the marginal tax benefit to the use of debt. An increase in the level of risk associated with the firm's cash flows causes the marginal probability of bankruptcy to increase, decreasing the marginal benefit and increasing the marginal cost of the use of debt. Obtaining the new optimum, therefore, entails a reduction in the use of debt. The economic intuition for the reason increased ambiguity results in higher optimal leverage is similar. An increase in ambiguity results in ambiguity averse investors underweighting the perceived probabilities of all states; with high cash flow states being underweighted more than are low cash flow states. This implies that the perceived marginal probability of bankruptcy decreases. Therefore, the marginal tax benefit increases and the marginal bankruptcy cost decreases when the level of ambiguity increases. An increase in leverage is, therefore, required to achieve the new optimum.

One view of the theoretical contribution of this analysis is that, perhaps for want of clear guidance regarding whether and how these different aspects of uncertainty provide distinct implications for corporate decision making, the existing literature has confounded risk and ambiguity. The analysis presented here highlights a sharp distinction in the impacts on the capital structure decision for the two components of uncertainty: ambiguity and risk. Importantly, the underlying decision theoretic framework provides a measure of ambiguity that can be estimated, allowing empirical examination of the separate roles ambiguity and risk play in the capital structure decision.

To examine the predicted positive relation between ambiguity and leverage, we investigate whether the firm's leverage ratio is related to the level of its ambiguity in standard pooled OLS regressions. In order to account for the unobservable firm specific component of leverage identified by Lemmon et al. (2008), we then transform the data by taking first differences of all the variables and consider the extent to which a change in ambiguity is followed by a change in leverage in the subsequent year. Finally, we employ a first differences two-stage least squares model, instrumenting for ambiguity, to provide further evidence of a causal relation between ambiguity and leverage. In each of the regressions, both lagged observations of a firm's realized ambiguity (computed from equity market data), as well as a measure of the ambiguity for an equivalent unlevered firm (to mitigate concern that computing ambiguity using equity return is a biased measure of firm ambiguity), are employed as the primary explanatory variables of interest. Book leverage and market leverage are both examined as dependent variables. In all but a single regression specification, the estimated coefficients are consistent with the positive relation between ambiguity and leverage predicted by the model, suggesting that ambiguity is an important explanatory variable for capital structure choice. Moreover, ambiguity is economically more significance than the explanatory variables for leverage commonly used in the literature.

Measuring ambiguity independently of ambiguity aversion, risk, and risk aversion is the main challenge in testing the predictions delivered by the theoretical model. The empirical measure of ambiguity is rooted in the decision theory framework of expected utility with uncertain probabilities (EUUP) (Izhakian, 2017). In that framework, aversion to ambiguity takes the form of aversion to mean-preserving spreads in probabilities. Thereby, the degree of ambiguity can be measured by the volatility of the *probabilities* of future outcomes, just as the degree of risk can be measured by the volatility of outcomes. The separation of risk and ambiguity is an important prerequisite for our empirical assessment of the impact of ambiguity on leverage.

We follow the methodology used by Izhakian and Yermack (2017), Brenner and Izhakian (2018), and Augustin and Izhakian (2019), and estimate firm-level ambiguity as the volatility of daily return probabilities estimated from intraday returns data. Brenner and Izhakian (2018), and Augustin and Izhakian (2019), at the market level and the firm level respectively, conduct extensive tests to allay concerns that this measure of ambiguity captures other well-known dimensions of uncertainty. Our own robustness tests similarly show that ambiguity is a distinct dimension of uncertainty in the context of the capital structure decision. The significant effect of our ambiguity measure on leverage is robust to the inclusion of many alternative uncertainty factors (e.g., volatility of return means, volatility of return volatilities, and disagreement among analysts) and market-microstructure factors (e.g., bid-ask spreads).

In a related study, Lee (2014) considers a similar question to that posed here and finds that increased ambiguity leads to a reduction in leverage. Lee, however, uses the smooth model of ambiguity aversion (Klibanoff et al., 2005) assuming a risk neutral utility function. Within such a model, ambiguity and preference for ambiguity are risk-dependent. Furthermore, in such a model, ambiguity and ambiguity aversion have the same mathematical structure as do risk and risk aversion in the standard model. It is, therefore, not surprising that his result mirrors the standard result for an increase in risk. Moreover, his model assumes that while the firm's manager is ambiguity averse, market participants are (and so market pricing is) ambiguity neutral. Lee's theoretical result, therefore, reflects an agency cost rather than value maximizing behavior, making it unclear how the model's predictions are expected to associate with observed firm behavior. Lee's empirical results are based on an event (the 1982 Voluntary Restraint Agreement on steel import quotas) that confounds risk and ambiguity, making interpretation of his estimates very difficult.[3]

The remainder of this paper is organized as follows. Section 2 presents a theoretical discussion of ambiguity and develops the model. Section 3 discusses the sample selection and empirical tests. Section 4 presents regression analysis of capital structure, and Section 5 concludes the paper. All proofs are provided in the Appendix.

2 The model

We develop a static model to examine the impact of aversion to ambiguity on corporate capital structure choice. The model has two important, distinguishing features. First, agents' preferences for

[3] Dahya et al. (2018) use an approach that is very similar to ours to study the implications of ambiguity for dividend payout policy decisions.

ambiguity are outcome-independent and therefore independent of preferences for risk. Second, agents are averse to ambiguity and to risk. The outcome-independence of agents' preferences for ambiguity is necessary to differentiate the effect of ambiguity from that of risk.

To provide a basic intuition for the effect of ambiguity and risk on the value of the firm (equity and debt), consider a simplified structural framework in which the optimal face value of the debt is F. If the firm's risk increases, as is illustrated in Panel (a) of Figure 1, the left-tail probability mass increases, increasing the marginal probability of default given the initial debt level F. This change increases the marginal bankruptcy cost and decreases the marginal tax benefit from debt financing at the original optimum F. As a result, optimal leverage is lower after the change.

A classic feature of many ambiguity models (e.g., Choquet expected utility, Schmeidler, 1989) is that ambiguity-averse investors act as if they underweight the perceived probabilities of states with high outcomes. Under Choquet expected utility in particular, they underweight the perceived probabilities of high outcome states to a greater extent than they underweight the perceived probabilities of low outcome states, such that probabilities are subadditive; i.e., probabilities sum up to a number smaller than one. In this case, as is illustrated in Panel (b) of Figure 1, in response to an increase in ambiguity, ambiguity averse agents perceive the left-tail probability mass decreases, decreasing the marginal probability of default at F. Therefore, the marginal bankruptcy cost falls and the marginal tax benefit rises at the original optimum and optimal leverage increases.

[Figure 1]

2.1 The decision theoretic framework

To develop a model of optimal capital structure with ambiguity and aversion to ambiguity, we employ the preference relation Choquet expected utility (CEU) (Schmeidler, 1989) and augment it with the theoretical framework contained in expected utility with uncertain probabilities (EUUP) (Izhakian, 2017).[4] In this context, EUUP can be viewed as providing an axiomatic development of the capacities (nonadditive probabilities) employed in the CEU model. EUUP derives these capacities based upon the ambiguity in the environment (beliefs) and the agents' aversion (attitude) to ambiguity. Under EUUP, preferences for ambiguity are outcome-independent, which allows for the separation of ambiguity from risk as well as the separation of attitudes from beliefs. Importantly, a by-product of this approach is a model-derived, risk-independent measure of ambiguity that is rooted in axiomatic decision theory

[4] The max-min expected utility model (MEU) of Gilboa and Schmeidler (1989) is a special case of CEU. However, MEU does not allow the separation of beliefs regarding ambiguity from attitudes toward ambiguity.

and can be employed to test the predictions of the theory.[5]

The main concept behind the CEU framework when augmented with EUUP is that the preferences for ambiguity are applied exclusively to the *uncertain* probabilities of future events. Thus, under the structure of EUUP, aversion to ambiguity is defined as an aversion to mean-preserving spreads in probabilities, which are outcome-independent. As such, the Rothschild and Stiglitz (1970) approach can be employed to measure ambiguity independently of risk, as the volatility of the uncertain probabilities of future events.

Formally, the investor, who values a risky and ambiguous payoff X, possesses a set $\mathcal{P}$ of prior probability distributions P over events, equipped with a prior probability ξ (a distribution over probability distributions). Each cumulative probability distribution $\mathrm{P} \in \mathcal{P}$ is associated with a marginal probability function $\varphi(\cdot)$. An investor evaluates the expected utility of a risky and ambiguous payoff by the CEU model (Schmeidler, 1989)

$$\mathrm{V}(X) \;=\; \int_{\mathcal{S}} \mathrm{U}(\cdot)\, d\mathrm{Q}, \tag{1}$$

where $\mathcal{S}$ stands for the state-space and Q for the capacity. Using the set of priors $\mathcal{P}$ and the second-order prior ξ, by EUUP, the investor assesses the capacities as the certainty equivalent probability of each event. A capacity, referred to in EUUP as the *perceived probability*, is the unique certain probability value that the investor is just willing to accept in exchange for the uncertain probability of a given event. Accordingly, the marginal perceived probability is assessed by[6]

$$d\mathrm{Q} \;=\; \mathrm{E}[\varphi(x)]\left(1+\frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(x)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(x)])}\mathrm{Var}[\varphi(x)]\right), \tag{2}$$

where the function Υ, called the *outlook function*, captures the investor's attitude toward ambiguity;[7] the expected marginal and cumulative probability of x are computed using ξ, such that $\mathrm{E}[\varphi(x)] \equiv \int_{\mathcal{P}} \varphi(x)\, d\xi$ and $\mathrm{E}[\mathrm{P}(x)] \equiv \int_{\mathcal{P}} \mathrm{P}(x)\, d\xi$; and $\mathrm{Var}[\varphi(x)] \equiv \int_{\mathcal{P}} \left(\varphi(x) - \mathrm{E}[\varphi(x)]\right)^2 d\xi$ defines the variance of the marginal probability. Using these perceived probabilities, the investor assesses the

[5]The measurement of ambiguity independently from risk poses a challenge for other frameworks which do not separate ambiguity from attitude toward ambiguity (e.g., Gilboa and Schmeidler, 1989) or in which preferences for ambiguity are outcome-dependent (e.g., Klibanoff et al., 2005; Chew and Sagi, 2008).

[6]Equation (2) is an approximation of the perceived probability $\mathrm{Q}(x) = \Upsilon^{-1}\left(\int_{\mathcal{P}} \Upsilon(\mathrm{P}(x))\, d\xi\right)$ (Theorem 2, Izhakian, 2018). The residual of the approximation is $R_2(\mathrm{P}(x)) = o\left(\mathrm{E}\left[|\mathrm{P}(x) - \mathrm{E}[\mathrm{P}(x)]|^3\right]\right)$ as $|\mathrm{P}(x) - \mathrm{E}[\mathrm{P}(x)]| \to 0$, which is negligible. Therefore, to simplify notation, we use the equal sign instead of the approximation sign.

[7]The outlook function is assumed to satisfy $\left|\frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}_X(x)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}_X(x)])}\right| \leq \frac{1}{\mathrm{Var}[\varphi_X(x)]}$., which bounds the concavity and the convexity of Υ to assure that the approximated marginal perceived probabilities are always positive and that they satisfy set monotonicity.

expected utility of a risky and ambiguous payoff $\mathrm{V}(X)$ by

$$\mathrm{V}(X) = \int \mathrm{U}(x) \underbrace{\mathrm{E}[\varphi(x)]\left(1+\frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(x)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(x)])}\mathrm{Var}[\varphi(x)]\right)}_{\text{Perceived Probability of Outcome } x} dx. \tag{3}$$

Risk aversion is (distinctly) captured by a strictly-increasing, concave, and twice-differentiable continuous utility function $\mathrm{U}:\mathbb{R}\to\mathbb{R}$, applied to the uncertain outcomes. As the investor is ambiguity-averse, she compounds the set of priors $\mathcal{P}$ using the second order prior ξ over $\mathcal{P}$ in a non-linear way. This aversion is captured by a strictly-increasing, concave, and twice-differentiable continuous function $\Upsilon:[0,1]\to\mathbb{R}$ applied to the probabilities.[8] The investor's (subadditive) perceived probabilities represented in Equation (3) are a function of the extent of ambiguity, measured by $\mathrm{Var}[\varphi(x)]$, and the investor's aversion to ambiguity, captured by $-\frac{\Upsilon''(\cdot)}{\Upsilon'(\cdot)}>0$. Both a higher aversion to ambiguity or a higher extent of ambiguity result in lower (more underweighted) perceived probabilities. When the investor is ambiguity neutral, Υ is linear, the perceived probabilities become the (additive) expected probabilities (the linear reduction of compound lotteries) and Equation (3) collapses to the standard expected utility framework. The same reduction of the model occurs when ambiguity is not present (there is no uncertainty over probabilities). To simplify the analysis and the discussion of the results, it is assumed that relative ambiguity aversion is nonincreasing, i.e., $\left(\frac{\Upsilon''}{\Upsilon'}\right)'\geq 0$. Many classes of outlook functions satisfy this condition, including constant relative ambiguity aversion (CRAA) and constant absolute ambiguity aversion (CAAA).

Based on the functional form in Equation (3), the degree of ambiguity can be measured by the expected probability weighted average (averaged across the relevant events) volatility of probabilities. Formally, the measure of ambiguity is given by

$$\mho^2[X] = \int \mathrm{E}[\varphi(x)]\,\mathrm{Var}[\varphi(x)]\,dx. \tag{4}$$

A major advantage of this measure is that it can be computed from data and employed in empirical tests. *Risk-independence* is another major advantage of $\mho^2$ (mho^2); unlike risk measures, it does not depend upon the magnitudes of the outcomes associated with the different events.

2.2 The asset pricing framework

We use a standard market structure framework, where the only variation in our analysis is the specification of probabilities. We assume markets are perfect and assume an absence of arbitrage opportunities (the law of one price holds). There are two dates, 0 and 1. The state at date 0 is known, and the

[8] Ambiguity aversion is reflected in the preference the investor has for the expectation of an uncertain probability over the uncertain probabilities. Recall that risk aversion is exhibited when an investor prefers the expected outcome of the uncertain outcome over the uncertain outcomes.

states at date 1 are ordered from the lowest consumption level to the highest. Throughout the analysis we assume a representative investor who is endowed with initial wealth w_0. Her wealth at time 1 is denoted w_1. Since, the asset pricing framework is used only to extract state prices, for simplicity, we assume a single product in the economy, with an uncertain payoff x at time 1. The investor's objective function can then be written in the usual way

$$\max_{c_0,\theta} \quad \mathrm{W}\left(c_0\right)+\mathrm{W}\left(c_1\right) \tag{5}$$

subject to the budget constraints

$$c_0 = w_0 - \theta \int q(x) x dx \qquad \text{and} \qquad c_1 = w_1 + \theta x,$$

where $q(x)$ is the date 0 price of a pure state contingent claim on state s with consumption x; and θ is the investor's position of the single asset. Using the functional form of expected utility in Equation (3), the state prices can be extracted as follows.

Theorem 1 *Suppose a time-separable utility function. The state price of state x is then*

$$q(x) \quad = \quad \pi\left(x\right) \frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}, \tag{6}$$

where

$$\pi\left(x\right) \quad = \quad \mathrm{E}\left[\varphi\left(x\right)\right]\left(1+\frac{\Upsilon''\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}{\Upsilon'\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}\mathrm{Var}\left[\varphi\left(x\right)\right]\right); \tag{7}$$

and $q(x)$ is unique and positive.

The state price $q(x)$ is the price of a claim to one unit of consumption contingent on the occurrence of state s (Arrow security).[9] The representation of state prices in Equation (6) illustrates the distinct impacts risk, ambiguity, and attitudes toward these aspects of uncertainty have on market pricing. Ambiguity and aversion to ambiguity impact state prices through the perceived probabilities, $\pi\left(x\right)$. Risk and aversion to risk impact the state price via the curvature of the utility function U and the magnitude of the outcomes x relative to consumption at time 0. When there is no ambiguity, i.e., probabilities are perfectly known, state prices $q(x)$ reduce to the conventional representation $q(x) = \varphi\left(x\right)\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}$.[10]

The following corollary is an immediate consequence of Theorem 1.

[9] Chapman and Polkovnichenko (2009) extract the state prices in a rank-dependent expected utility framework.

[10] In the presence of ambiguity with ambiguity neutral investors state prices are $q(x) = \mathrm{E}\left[\varphi\left(x\right)\right]\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}$, consistent with the conventional representation.

Corollary 1 *The risk neutral probability of state x is*

$$\pi^*(x) \;=\; \frac{q(x)}{\displaystyle\int q(x)dx} \;=\; \frac{\pi\left(x\right)\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}}{\displaystyle\int \pi\left(x\right)\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}dx}, \tag{8}$$

and the risk-free rate of return is

$$r_f \;=\; \frac{1}{\displaystyle\int q(x)dx} - 1 \;=\; \frac{1}{\displaystyle\int \pi\left(x\right)\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}dx} - 1. \tag{9}$$

Note that while they are dependent upon the subadditive perceived probabilities, the risk neutral probabilities are additive. Similarly to the state prices, ambiguity and aversion to ambiguity transform the standard representation of risk neutral probabilities via the $\pi\left(x\right)$ term.

2.3 The capital structure decision in a perfect capital market

The capital structure model developed here is based on the canonical tradeoff theory model of Kraus and Litzenberger (1973). Consider a one-period project that requires a capital investment I at time 0. The payoff of this project, obtained at time $t = 1$, is a risky and ambiguous variable. As discussed above, it is assumed that the owner of the project is a representative investor, whose set of priors, the set of possible probability distributions $\mathcal{P}$ for X, is identical to the set of priors of each investor in the economy, and her beliefs (second-order probability distribution) over $\mathcal{P}$ is identical to that of each investor.

At time $t = 0$, a decision to invest in a project is made if and only if

$$\frac{1}{1+r_f}\mathbb{E}^*\left[X\right] > I,$$

where $\mathbb{E}^*$ is the expectation taken with respect to the risk neutral probabilities, π^*, defined in Corollary 1.[11] Given the decision to invest in the project, its owner must decide at time $t = 0$ what mix of debt and equity financing the firm will use to establish the project. The objective of this decision is to maximize the value of the firm:

$$\begin{aligned}
\max_F \qquad & S_0\left(F\right) + D_0\left(F\right) \\
\text{s.t.} \quad S_0 \; & = \frac{1}{1+r_f}\mathbb{E}^*\left[\max\left(X - F\ ,\ 0\right)\right] \\
D_0 \; & = \frac{1}{1+r_f}\mathbb{E}^*\left[\min\left(F\ ,\ X\right)\right],
\end{aligned}$$

where S_0 is the market value of the common shares; D_0 is the market value of debt; and F is the debt's face value. Without loss of generality, the debt is assumed to be a one-period zero-coupon bond. The

[11] The double-struck capital font is used to designate expectation or variance of outcomes, taken with respect to the expected probabilities or with respect to risk-neutral probabilities, while the regular straight font is used to designate expectation or variance of probabilities, taken with respect to the second-order probabilities, ξ.

optimization problem can be written more explicitly as

$$\begin{aligned}
\max_{F} \quad & S_0(F) + D_0(F) \\
\text{s.t.} \quad S_0 &= \frac{1}{1+r_f} \int_F^{\infty} \pi^*(x)\,(x - F)\,dx \\
D_0 &= \frac{1}{1+r_f} \left(\int_0^F \pi^*(x) x dx + \int_F^{\infty} \pi^*(x) F dx \right).
\end{aligned}$$

Any $x < F$ is considered a default state.

Theorem 2 *Suppose that there are no taxes, no bankruptcy costs and no asymmetric information. Modigliani-Miller's capital structure irrelevance proposition is maintained when all agents in the economy are averse to ambiguity.*

Proposition 1 in Modigliani and Miller (1958) shows that, in an economy without taxes, bankruptcy costs or asymmetric information, the capital-structure choice of a firm is irrelevant to its market value. Their proof is based on the idea the an investor can generate the return of holding the levered firm's equity by holding the equity of an equivalent unlevered firm and a loan in the same proportion as the leverage ratio of the levered firm. The same argument holds true in an ambiguous economy in which all decision makers are ambiguity averse.

2.4 The over-investment problem

In their seminal paper, Jensen and Meckling (1976) introduce the "over-investment" problem. Central to their idea is the simple notion that in the equity holders' view, levered equity can be described as a call option, where the exercise price is the face value of the debt (e.g., Merton, 1974). Maintaining the assumptions of an absence of taxes and bankruptcy costs, in the present model, the value of the equity is the expected value of the future equity payoff (using the risk neutral probabilities) discounted at the risk free rate,

$$S_0 = \frac{1}{1+r_f} \int_F^{\infty} \pi^*(x)\,(x - F)\,dx. \tag{10}$$

This value is affected by ambiguity through $\pi^*(x)$ and by risk through the magnitude of x.

As in Rothschild and Stiglitz (1970), we consider an asset as becoming risker if its payoffs at time t can be written as a mean-preserving spread of its payoff at time $t-1$. The following proposition establishes that the standard relation between the value of levered equity and the level of risk holds true in our setting.

Proposition 1 *For a given face value of debt, the higher is the degree of risk, the higher is the value of the levered equity.*

Levered equity holders benefit from the upside risk and are protected from downside risk. The value of levered equity is, therefore, increasing in the risk of the underlying cash flow and levered equity holders have an incentive to increase the risk of the firm's cash flow. The over-investment problem follows from the recognition that levered equity holders, therefore, benefit from an increase in risk even at the expense of some reduction in the value of the cash flow. Namely, levered equity holders benefit if the firm undertakes a negative net present value investment opportunity when the investment provides a sufficient increase in the risk of the firm's cash flow. The next proposition shows that this incentive becomes more complex in a setting with ambiguity and ambiguity averse investors. In Proposition 2, an increase in ambiguity occurs when the possible probabilities associated with a given outcome at time t can be written as a mean preserving spread of the possible probabilities at time $t-1$

Proposition 2 *For a given face value of debt, if the degree of ambiguity increases, the value of the levered equity is reduced.*

The intuition for this result is equally straightforward. An increase in the level of ambiguity causes ambiguity-averse investors to further underweight the probabilities of "good" outcomes, reducing the value of the option like payoff represented by levered equity. Similarly, as investors become more averse to ambiguity, the value of levered equity will fall. It is important to note that it is the combination of ambiguity and individuals' aversion to ambiguity that causes the change in perceived probabilities. If individuals are ambiguity neutral, an increase in ambiguity has no impact on the value of levered equity.

Levered equity holders, therefore, see a loss in value from an increase in ambiguity. With ambiguity and aversion to ambiguity, there exists a three way tradeoff between value, risk, and ambiguity. The incentive of levered equity holders regarding an investment that increases uncertainty (the combination of risk and ambiguity) is, therefore, unclear. An often cited example of the over-investment problem is the incentive of a firm's managers to pursue firm value reducing M&A activities due to their potential to increase the risk of the firm's cash flow. However, it is plausible that while a given merger may increase the level of risk of a firm's cash flow, the merger will also increase the level of ambiguity associated with the firm's cash flow. In such a case, the incentive of a manager acting in the interests of the levered equity holders is unclear. A variety of outcomes are possible. At one extreme, the manager may avoid a merger that would increase firm risk and firm value if it will increase ambiguity significantly, implying an "under-investment" problem. A deeper examination of the effect of ambiguity on M&A

decisions is left for future research.

A related result is provided in Garlappi et al. (2017). They propose that heterogeneous beliefs among a corporate board or management team can be modeled as decision-making group (DMG) that possesses a set of priors. In that case, the DMG can be viewed as facing ambiguity, à la the multiple prior paradigm (Gilboa and Schmeidler, 1989). Therefore, the greater is the disagreement (or the more heterogenous are the beliefs) amongst the DMG, the higher is the ambiguity or the aversion to ambiguity of the "corporation." When the DMG must decide whether to invest in a new project, Garlappi, Giammarino, and Lazrak show that, while individuals within the group may independently believe that the investment provides value, collectively, the group may decide not to invest.

2.5 Taxes and bankruptcy costs

Consider now an economy with corporate taxes and bankruptcy costs. Recall that any state $x < F$ is considered a default state. Bankruptcy costs are assumed to be proportional to the firm's output in the event of default

$$B(x;F) = \begin{cases} \alpha x, & x < F \\ 0, & x \geq F \end{cases},$$

where $0 < \alpha < 1$. The tax benefit associated with the use of debt is, for simplicity, based on the entire debt service in non-default states,[12]

$$T(x;F) = \begin{cases} 0, & x < F \\ \tau F, & x \geq F \end{cases},$$

where τ is the corporate income tax rate. In such an economy, firm value is a function of the chosen capital structure. In particular the amount of debt F issued can be written

$$\begin{aligned} V(F) &= S_0(F) + D_0(F) \\ &= \frac{1}{1+r_f}\left(\int_F^\infty \pi^*(x)(1-\tau)(x-F)\,dx + \int_0^F \pi^*(x)(1-\alpha)\,x dx + \int_F^\infty \pi^*(x) F dx\right). \end{aligned} \tag{11}$$

To maximize the firm's value, the capital structure choice problem can be written

$$\begin{aligned} \max_F \quad & S_0(F) + D_0(F) \\ \text{s.t.} \quad S_0(F) &= \frac{1}{1+r_f}\mathbb{E}^*\left[\max\big((1-\tau)x - F + T(x;F)\ ,\ 0\big)\right] \\ D_0(F) &= \frac{1}{1+r_f}\mathbb{E}^*\left[\min\left(F\ ,\ x - B(x;F)\right)\right]. \end{aligned}$$

[12] See, for example, Kraus and Litzenberger (1973).

Notice that S_0 can be written

$$S_0(F) = \frac{1}{1+r_f}\mathbb{E}^*\left[\max\left((1-\tau)(x-F)\ ,\ 0\right)\right].$$

With the capital structure choice problem defined above, the next theorem identifies the optimal level of leverage for the firm.

Theorem 3 *The optimal leverage satisfies*

$$F = \frac{\tau}{\pi^*(F)\alpha}\int_F^\infty \pi^*(x)dx. \tag{12}$$

Alternatively, the optimal leverage can be written in terms of a risk neutral hazard rate for default. It can be immediately observed from Equation (12) that a higher tax rate, τ, or a lower bankruptcy cost, α, positively affect the use of leverage. Our focus is, however, on the implications of changes in ambiguity and risk for capital structure choice. Proposition 3 describes the consequence of a comparative static change in risk.

Proposition 3 *The higher is the degree of risk, the lower is the optimal leverage.*

Proposition 4 describes the consequence of a comparative static change in ambiguity.

Proposition 4 *The higher is the degree of ambiguity, the higher is the optimal leverage.*

Propositions 3 and 4 demonstrate that increased risk has a *negative* effect on the chosen level of leverage, while increased ambiguity has a *positive* effect. The intuition for the result in Proposition 3 is standard: greater volatility in the project payoff has a symmetric effect, pushing probability mass from the center of the distribution to its tails. The increase in volatility has the effect of increasing the perceived marginal bankruptcy cost and decreasing the perceived marginal tax benefit. This effect is illustrated in Panel (a) of Figure 1. A reduction in the use of debt, to F', is required to once again equate the cost and benefit of the use of debt financing at the margin.

It can be tempting to suggest that this intuition can be applied to an increase in ambiguity as well; more ambiguity or uncertainty regarding future cash flow might be thought to have the same effect. An increase in ambiguity, however, represents an increased uncertainty regarding probabilities. This increased ambiguity is independent of the outcomes (unlike an increase in risk), which implies the standard intuition cannot be applied. A more appropriate intuition derives from the effect of ambiguity when investors are ambiguity averse. Ambiguity averse investors underweight the probabilities; they underweight the probabilities of high output states to a greater extent than they do the probabilities of low output states. Therefore, an increase in ambiguity has an asymmetric effect on the perceived probability distribution, as illustrated in Panel (b) of Figure 1. The *total* bankruptcy cost and the *total* value of the tax shields are both reduced as a result of this change. An increase in ambiguity decreases the marginal perceived probability of default at the original optimum (*i.e.* the marginal perceived probability of $x = F$). Conversely to the standard intuition for an increase in risk, this implies that the marginal bankruptcy cost at the original optimum is reduced and that the marginal tax benefit associated with debt financing is increased. An increase in leverage is required to attain the new optimum.

3 Empirical design

We turn now to test the predictions of Propositions 3 and 4 empirically. For consistency with the majority of the empirical capital structure literature, we examine how annual observations of firms' leverage ratios are related to annual observations of ambiguity and cash flow volatility. Similar results are obtained using quarterly data.

3.1 Sample selection

We estimate the relation between ambiguity and leverage using a sample of all nonfinancial firm-year observations in the annual Compustat database between 1993-2017. The time window 1993-2017 is dictated by the intraday stock data available on the TAQ database. This data is used in estimating the degree of ambiguity associated with each firm. After a process of data cleaning and filtering described below, we analyze $47,915$ annual observations for $4,242$ unique firms over the 25 years between 1993 and 2017.

To construct our sample, we begin with all records from the Compustat fundamentals annual database. We drop all financial firms as well as all government entities. We also drop all observations missing one of the following descriptors: fiscal year end (FYR), total assets (AT), debt in current liabilities (DLC), long-term debt (DLTT) or stock price (PRCC). Finally, we drop all observations with negative values for sales, total assets or debt, and all observations with a negative leverage ratio or leverage ratio greater than 100% (both book or market ratios). In addition, we drop all duplicate records or records for which we are unable to match identifiers to the CRSP or TAQ stock price databases. All observations for which the annual degree of ambiguity, risk or cash flow volatility cannot be estimated are also dropped. To avoid potential biases that might be caused by outlier observations, for each continuous variable, 1% of observation with outlier values (highest 1% for variables taking only positive values and in addition lowest 1% for variables taking any values) are dropped, which leaves us with $47,915$ observations.

We estimate the annual degree of ambiguity, risk and cash flow volatility for each firm and each year as detailed below. For ambiguity and risk, we estimate the monthly values and then use their average over the fiscal year as the annual ambiguity and risk associated with each firm-year observation. As described below, firm levered and unlevered measures of ambiguity and risk are computed to examine whether the use of equity based measures are responsible for the reported findings.

3.2 Estimating ambiguity

The main motivation for our use of the CEU model augmented with the EUUP framework is that Equation (3) naturally implies a risk-independent measure of ambiguity, denoted by $\mho^2$. Using the EUUP framework, the degree of ambiguity can be measured by the volatility of uncertain *probabilities*, just as the degree of risk can be measured by the volatility of uncertain *outcomes*. Formally, the measure of ambiguity is defined by Equation (4), which represents an expected probability weighted average of the variances of probabilities. We follow Izhakian and Yermack (2017) and Augustin and Izhakian (2019), and estimate the monthly degree of ambiguity for each firm using intraday stock return data from the TAQ database.[13]

The challenge in estimating ambiguity as identified in Equation (4) (or the implementation version in Equation (13) below) is to measure the expectation of and the variation in probabilities across the set of possible prior probability distributions. Using the representative investor assumption, each prior in the set is assumed to be represented by the observed daily intraday returns on the firm's (levered or unlevered) equity, and the number of priors in the set is assumed to depend on the number of trading days in the month. The set of priors thus consists of 18-22 realized distributions over a month. For practical implementation, we discretize return distributions into n bins $B_j = (r_j, r_{j-1}]$ of equal size, such that each distribution is represented as a histogram, as illustrated in Figure 2. The height of the bar of a particular bin is computed as the fraction of daily intraday returns observed in that bin, and thus represents the probability of the outcomes in that bin. Equipped with these 18-22 daily return histograms, we compute the expected probability of being in a particular bin across the daily return distributions for each month, $\mathrm{E}\left[\mathrm{P}\left(B_j\right)\right]$, as well as the variance of these probabilities, $\mathrm{Var}\left[\mathrm{P}\left(B_j\right)\right]$. To this end, an equal likelihood is assigned to each histogram.[14] Using these values, the monthly degree

[13]The measure of ambiguity, defined in Equation (4), is distinct from aversion to ambiguity. The former, which is a matter of beliefs (or information), is estimated from the data, while the latter is a matter of subjective attitudes and endogenously determined in the empirical tests.

[14]This is consistent with the *principle of insufficient reason*, which states that given n possibilities that are indistinguishable except for their names, each possibility should be assigned a probability equal to $\frac{1}{n}$ (Bernoulli, 1713; de Laplace, 1814). It is also consistent with the idea of the simplest non-informative prior in Bayesian probability (Bayes et al., 1763), which assigns equal probabilities to all possibilities; and the principle of maximum entropy (Jaynes, 1957), which

of ambiguity of firm i is then computed as follows:

$$\mho^2[r_i] \equiv \frac{1}{\sqrt{w(1-w)}} \sum_{j=1}^{n} \mathrm{E}[\mathrm{P}_i[B_j]] \operatorname{Var}[\mathrm{P}_i[B_j]] . \tag{13}$$

To minimize the impact of the selected bin size on the value of ambiguity, we apply a variation of Sheppard's correction and scale the weighted-average volatilities of probabilities to the size of the bins by $\frac{1}{\sqrt{w(1-w)}}$, where $w = r_{i,j} - r_{i,j-1}$.

[Figure 2]

In our implementation, we sample five-minute stock returns from 9:30 to 16:00, as this eliminates microstructure effects (Andersen et al., 2001). Thus, we obtain daily histograms of up to 78 intraday returns. If we observe no trade in a specific time interval for a given stock, we compute returns based on the volume-weighted average of the nearest trading prices. We ignore returns between closing and next-day opening prices to eliminate the impact of overnight price changes and dividend distributions. We drop all days with less than 15 different five-minute returns; we also drop months with less than 15 intraday return distributions. In addition, we drop extreme returns (plus or minus 5% log returns over five minutes), as many such returns are due to improper orders that are subsequently canceled by the stock exchange.[15]

For the bin formation, we divide the range of daily returns into 162 intervals. We form a grid of 160 bins, from -40% to 40%, each of width 0.5%, in addition to the left and right tails, defined as $(\infty, -40\%]$ and $(+40\%, +\infty)$, respectively. We compute the mean and the variance of probabilities for each interval, assigning equal likelihood to each distribution (i.e., all histograms are equally likely).[16] Some bins may not be populated with return realizations, which makes it difficult to compute their probability. Therefore, we assume a normal return distribution and use its moments to extrapolate the missing return probabilities. That is, $\mathrm{P}_i[B_j] = \left[\Phi(r_j; \mu_i, \sigma_i) - \Phi(r_{j-1}; \mu_i, \sigma_i)\right]$, where $\Phi(\cdot)$ denotes the cumulative normal probability distribution, characterized by its mean μ_i and the variance σ_i^2 of the returns. As in French et al. (1987), the variance of the returns is computed by applying the adjustment for non-synchronous trading, as proposed by Scholes and Williams (1977).[17]

states that the probability distribution which best describes the current state of knowledge is the one with the largest entropy.

[15]Our results are robust to the inclusion of extreme price changes, as well as for cutoffs at lower levels of 1% in terms of log returns.

[16]The assignment of equal likelihoods is equivalent to assuming that the daily ratios $\frac{\mu_j}{\sigma_j}$ are Student's-t distributed. When $\frac{\mu}{\sigma}$ is Student's t-distributed, cumulative probabilities are uniformly distributed (e.g., Proposition 1.27, page 21 Kendall and Stuart, 2010).

[17]Scholes and Williams (1977) suggest adjusting the volatility of returns for non-synchronous trading as $\sigma_t^2 =$

An important characteristic of the measure of ambiguity implied by EUUP is that it is outcome-independent (up to a state-space partition), which allows for a risk-independent examination of the impacts of ambiguity on financial decisions. Other proxies for ambiguity that have been used in the literature for empirical applications include the volatility of the mean return (Franzoni, 2017), the volatility of return volatility (Faria and Correia-da Silva, 2014), or the disagreement of analyst forecasts (Drechsler, 2013). As these measures are sensitive to changes in the set of outcomes (i.e., are outcome-dependent), they are risk-dependent and therefore less useful for this study. These proxies are, therefore, conceptually different and only weakly related to our measure of ambiguity. Similarly, skewness, kurtosis (as well as other moments of the return distribution) and $\mho^2$ are also conceptually different, as the former are outcome-dependent and the latter is outcome-independent. Jumps, time varying mean and time varying volatility are also outcome-dependent and conceptually different from ambiguity.

Brenner and Izhakian (2018) and Augustin and Izhakian (2019) study the implications of ambiguity for the equity premium and for spreads of credit default swaps. Their results indicate that in these contexts $\mho^2$ does not simply reflect other well-known "uncertainty" factors including skewness, kurtosis, variance of variance, variance of mean, downside risk, mixed data sampling measure of forecasted volatility (MIDAS), jumps, or investors' sentiment, among many others. Their tests also help to mitigate the concern that observed returns are generated by a single (additive) probability distribution. In our robustness tests, we also examine many of these uncertainty factors at the firm level. In particular, in Table 8 we examine the explanatory power of our measure of ambiguity relative to others that have been proposed, and show that ambiguity represents a distinct aspect of uncertainty.

It has long been recognized that leverage magnifies measures of volatility, creating cross-sectional variation in volatility that is related to leverage. It is possible that leverage may affect the measure of ambiguity, however, due to the outcome independence of ambiguity the nature of any such impact is unclear. Furthermore, differencing the data will remove some of any cross-sectional explanatory power of ambiguity created by leverage and so we do not predict that this will have a material impact on the reported results. Nevertheless, for completeness, we estimate all regressions using both a measure of ambiguity based the firm's levered returns and on a proxy for its unlevered returns. This is done by combining the book value of total debt and the market value of equity to represent firm value for every

$\frac{1}{N_t}\sum_{\ell=1}^{N_t}\left(r_{t,\ell}-\mathrm{E}\left[r_{t,\ell}\right]\right)^2+2\frac{1}{N_t-1}\sum_{\ell=2}^{N_t}\left(r_{t,\ell}-\mathrm{E}\left[r_{t,\ell}\right]\right)\left(r_{t,\ell-1}-\mathrm{E}\left[r_{t,\ell-1}\right]\right)$. We perform all estimations without the Scholes-Williams correction for non-synchronous trading. The results are essentially the same.

five-minute interval.[18] The resulting periodic firm values are used to estimate unlevered or firm level returns for each interval. Unlevered ambiguity is computed from these unlevered returns as described above.

For consistency, we compute risk using the same five-minute returns that we use to measure ambiguity. For each individual stock i on each day, we compute the variance of intraday returns, applying the Scholes and Williams (1977) correction for non-synchronous trading and a correction for heteroscedasticity.[19] In a given year, we then compute the annual variance of stock returns using the average of daily variances, scaled to a monthly frequency.[20]

3.3 Leverage and firm specific characteristics

The annual leverage ratio of each firm is the main dependent variable in our empirical tests; and we employ both the book value and the market value versions of this ratio. Book leverage is computed as "debt in current liabilities" plus "long-term debt" divided by the total book value of assets. Market leverage is computed as "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt."

In addition to the leverage ratios, we obtain a set of annual firm characteristics using variables that are standard in the empirical capital structure literature. We compute the cash flow volatility by taking the variance of quarterly cash flows over the fiscal year. For convenience, we normalize the cash flow volatility by 1,000,000. Firm size is measured by the log of sales normalized by gross domestic product (GDP).[21] Firm profitability is measured by operating income before depreciation divided by book assets. Asset tangibility is measured by property, plant and equipment divided by book assets. The market to book ratio is measured by the market value of equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is measured by R&D expenses relative to sales normalized by $10,000$. In addition, to explore the firms' leverage ratios relative to their industry, we also include industry median annual leverage, where firms are classified by their four-digit SIC codes. Expected marginal tax rates are obtained from John Graham's website. When the expected marginal tax rate is missing, we use the industry median annual marginal tax rate if available or the market-wide median annual marginal tax

[18] We consider three different frequencies of estimating the market value of the firm: updating the value every five-minutes, every day, and every month. Our results are robust across these alternatives.

[19] See, for example, French et al. (1987).

[20] We also estimate risk as the average monthly variance of daily returns. The results are essentially the same.

[21] Alternatively, firm size can be measured by the log of book assets normalized to the annual GDP level. We test all predictions using this alternate measure of firm size and the results are virtually identical.

rate if necessary. Finally, firm age is measured relative to the first year the stock price is reported in Compustat (Hadlock and Pierce, 2010).

3.4 Statistics

Table 1 presents summary statistics for the key variables used in the paper. Based on the statistics presented in the table, our sample is broadly consistent with those from other recent studies.

[Table 1]

Table 2 presents a correlation matrix for the variables. The two measures of uncertainty, cash flow volatility and ambiguity, are only weakly related to one another with an estimated correlation of 0.051 reported in Table 2. This suggests that the two variables capture economically distinct aspects of financial uncertainty, a conclusion that coincides with the comprehensive tests of Brenner and Izhakian (2018) and Augustin and Izhakian (2019).

The positive correlations between ambiguity (or unlevered ambiguity) and the firm characteristics size (sales) and age are important to consider as they may appear counter-intuitive. The degree of ambiguity within the EUUP model, $\mho^2$, is measured as the variance of the possible probabilities for each event, averaged across the possible events.[22] Therefore, $\mho^2$ of a given firm is based upon the nature of the set of possible probability distribution govern its outcomes characterizing the firm and the distribution of these distributions (the second-order distribution). Large or old firms may have a "narrower" set of possible distributions that may govern their outcomes, but these distributions have high precision (i.e., are steeply sloped and therefore less risky), such that, across the distributions, each possible outcome has a wide variety of possible probabilities associated with it. This implies that the variance of the probabilities (ambiguity) of each outcome is high. In contrast, small or young firms may have a "wider" set of possible distributions that may govern their outcomes, but these distributions have low precision (are not steeply sloped and therefore more risky), such that each possible outcome has a narrow set of possible probabilities associated with it. In turn, this implies that the variance of the probabilities of each outcome is low and so is ambiguity.[23]

[22] The max-min model (Gilboa and Schmeidler, 1989) has established a common intuition for the level of ambiguity as related to the probability distribution that generates the worst-case scenario. However, rather than provide a useful framework for evaluating the degree of ambiguity this actually points out a limitation of the max-min model for evaluating the extent of ambiguity. As an extreme example, consider an Ellsberg urn experiment in which the decision maker is faced with an urn from which to draw a ball. A black ball results in a payoff of \$1 and a red ball a payoff of \$0. The decision maker is told that the urn has 100 total balls, all of which are either black or red, and that the number of black balls is either 20 or 80. Contrast that experiment with one in which the decision maker is told that the number of black balls is either 20 or 21. The decision maker clearly faces more uncertainty over probability distributions (greater ambiguity) in the first experiment, however the max-min model views the two scenarios identically.

[23] In the extreme case the distributions tend to the uniform distribution and therefore ambiguity tends to zero,

It seems intuitive to suggest that there would be greater ambiguity associated with small or young firms than with large or old firms. However, the intuition described above suggests the opposite, which can help explain the positive correlations in Table 2 between ambiguity and firm size or firm age. The relation between ambiguity, risk and firm size is examined more completely in the Appendix A.1. Note also that in all tests, firm size and firm age are included as control variables so that the estimated relation between leverage and ambiguity holds firm specific characteristics constant.

[Table 2]

3.5 Regression tests

To test the prediction of Proposition 4, we use three empirical models. The first model explores firm-level variation in the level of the leverage ratio L. This model uses the time series and cross sectional regression that is standard in the empirical capital structure literature.

$$L_{i,t} \quad = \quad \alpha + \beta \cdot \mho^2_{i,t-1} + \gamma \cdot Z_{i,t-1} + \varepsilon_{i,t}, \tag{14}$$

where i designates firm and t designates year. The vector $Z_{i,t-1}$ consists of the standard controls in the literature for firm characteristics as described in Section 3.3, including cash flow volatility at the firm level as the relevant measure of risk.[24] The main goals of this regression test are to examine the explanatory power (beyond the standard control variables) provided by ambiguity for the leverage ratio and to provide a benchmark for comparison to the existing literature.

The second model utilizes first differences of the dependent and independent variables to examines the effect of firm-level change in ambiguity on the change in the firm's leverage ratio. To this end, we estimate

$$\Delta L_{i,t} \quad = \quad \alpha + \beta \cdot \Delta\mho^2_{i,t-1} + \gamma \cdot \Delta Z_{i,t-1} + \varepsilon_{i,t}, \tag{15}$$

where $\Delta L_{i,t}$ is the change in variable L between time $t-1$ and t, and $\Delta Z_{i,t-1}$ is the change in the vector of variables Z between time $t-2$ and time $t-1$. This test is introduced in order to address the omitted variables problem inherent in the standard levels regression, as identified by Lemmon et al. (2008). Lemmon et al. employ firm fixed effects to control for the omitted variables bias. DeAngelo and Roll (2015) provide evidence to suggest that the unobserved heterogeneity in leverage is not time invariant. Furthermore, Lemmon et al. note that use of firm fixed effects "*sweeps out all of the cross sectional variation so that the model cannot identify what is responsible for the majority of the variation in*

[24] The relationship between risk and leverage has been examined in a host of studies (*e.g.* Titman and Wessels (1988) and Frank and Goyal (2009)) so we focus on Proposition 4 in our empirical analysis.

leverage ratios." Because we expect both cross sectional and time series variation in ambiguity to be important in explaining leverage, we employ the use of first differences of the variables rather than firm fixed effects to address this issue.

Finally, to provide further support for a causal relation between ambiguity and leverage, we conduct a first difference 2-Stage least squares regression. In the first stage, we estimate the change in ambiguity using two instrumental variables. To proxy for firm level ambiguity, we use the number of lawsuits in which the firm is named as a defendant in a particular year (DEF). To proxy for economy wide ambiguity, we use an indicator of whether the US house and the senate are controlled by the same political party as that of the president (CTL), i.e., a "unified government." In particular, we regress the change in ambiguity at time t on DEF_t, CTL_t and the changes in the other control variables

$$\Delta\mho^2_{i,t} \quad = \quad \alpha + \beta_1 \cdot CTL_t + \beta_2 \cdot DEF_{i,t} + \gamma \Delta Z_{i,t} + \epsilon_{i,t}, \tag{16}$$

and use the coefficients of this regression to estimate $\widehat{\Delta\mho}^2_{j,t}$. Then, the second stage of the model estimates the effect of the fitted change in ambiguity on the change in leverage by

$$\Delta L_{i,t} \quad = \quad \alpha + \beta \cdot \widehat{\Delta\mho}^2_{i,t-1} + \gamma \cdot \Delta Z_{i,t-1} + \varepsilon_{i,t}. \tag{17}$$

The different regression models are tested using both book leverage and market leverage. Each model is also tested using both levered and unlevered ambiguity at the firm level. All reported standard errors are clustered by firm and are robust to within firm heteroskedasticity.

4 Empirical findings

4.1 Main findings

First, we test whether the level of the leverage ratio is affected by the level of ambiguity in the previous year, as detailed in the basic model in Equation (14). Table 3 presents the findings of these regression tests. Panel A shows estimates for the book leverage ratio, while Panel B shows estimates for the market leverage ratio. In both panels, the value of ambiguity is used in the first four columns, while unlevered ambiguity, as defined earlier, is used in the right four columns. In each case, we present estimates for leverage as a function of ambiguity alone, then as a function of risk (cash flow volatility) alone, then in a model with both ambiguity and risk included, and finally in a model that also includes a set of commonly used control variables.

Table 3 demonstrates a relatively consistent pattern of results. When ambiguity or unlevered ambiguity are included in the model alone or alongside other variables, the estimated coefficients are almost uniformly positive and highly significant, consistent with the hypothesized relation between ambiguity and leverage. The sole contrasting result is reported in column 4 of Panel B, where ambiguity is negatively related to the market leverage ratio when the complete set of control variables are included in the regression. When cash flow volatility is used in the model, alone or with an ambiguity measure, the estimated coefficient is usually positive and significant, contrary to the standard hypothesis (an exception is panel A column 7, when cash flow volatility and unlevered ambiguity are the explanatory variables, the estimated coefficient on cash flow volatility is negative but insignificant). When cash flow volatility is used in the full model (columns 4 and 8 in panels A and B) the estimated coefficient on cash flow volatility becomes negative and significant consistent with the standard hypothesis.

[Table 3]

Next, we estimate a model designed to control for the endogeneity problem inherent in the standard levels regression by taking first differences of the variables and testing whether the change in leverage is explained by the lagged change in ambiguity. Namely, we test the model in Equation (15), which suggests that a change in ambiguity will be followed by a change in the use of leverage in the subsequent year. The findings are reported in Table 4. As with the level regressions, we test both book leverage (Panel A) and market leverage (Panel B). Estimates are arranged identically to those in Table 3. In these tests, we find that in every model, the ambiguity (or unlevered ambiguity) variable has a positive and statistically significant coefficient estimate. In the regression presented in panel A (book leverage), the coefficient estimates for cash flow volatility are all essentially zero and insignificant. However for the regression in panel B (market leverage) the estimated coefficients on cash flow volatility all negative and most are statistically significant.[25]

The control variables in Tables 3 and 4 generally have estimates in line with prior studies. Leverage tends to be higher in larger firms and for firms with more tangible assets, while more profitable firms and firms with higher market to book ratios tend to use less leverage. When using the first differences of explanatory variables to explain the change in book leverage, only the coefficient estimates for asset tangibility are significant, while in the market leverage changes regression and in the levels regressions more of the estimated coefficients are statistically significant. One surprising result in our empirical model is that the estimated relation between research & development spending and leverage is essentially zero. Most studies (e.g., Mackie-Mason, 1990; Berger et al., 1997) find a negative relation, in line with the Myers (1977) prediction that growth opportunities will be financed primarily by equity.

[Table 4]

[25]Note that in these differences regression tests, firm age is not included as a control variable in the changes regression for the obvious reason.

Note that when we estimate the regression using changes, the estimates for the ambiguity variable—the central focus of our study—are positive and significant regardless of the controls used or the inclusion or exclusion of the cash flow volatility variable. For all specifications of the regressions the estimates for the effect of ambiguity on leverage appear to be more consistent and more reliably significant than the estimates for the risk variable. Using firm and year fixed effects rather than first differences to control for the firm specific unobserved heterogeneity in leverage provides similar results. Specifically, the (unreported) estimated coefficients on ambiguity or unlevered ambiguity are positive and significant while the estimated coefficients on cash flow volatility vary depending on the model.

To ensure that the results are not driven by the smallest firms in our sample, we estimate the regressions in Table 4 on a sample that excludes the firm year observations for the smallest decile of firms in each year. The (unreported) results are very similar to those reported in Table 4. In all the models the estimated coefficients for ambiguity or unlevered ambiguity are positive and significant. Our findings are also robust across a variety of specifications for the sample of firms included in the tests. If we trim rather than winsorize the data, require consecutive data points to ensure a time series component to the estimates, use quarterly rather than annual data, use the variance of equity returns to measure risk, or measure firm size using the natural log of book assets rather than sales, the effect of ambiguity is consistently positive and significantly related to leverage while the relation between cash flow volatility and leverage varies depending upon the regression specification.[26] The somewhat erratic significance of our risk variable is consistent with Frank and Goyal (2009), who find that risk, measured as the variance of equity returns, is not one of the "core factors" determining leverage, although their analysis is based on the incremental contribution to R-squared rather than on the economic or statistical significance of coefficient estimates.

4.2 Instrumental variable regression findings

To provide further evidence regarding a causal relation between ambiguity and leverage, Table 5 presents the findings of a first difference 2-Stage least squares estimate of the model in Equations (16) and (17). Specifically, in the first stage, we instrument for the first difference in the ambiguity measures using two proxies for ambiguity: the number of lawsuits in which the firm is named as a defendant in each year as a proxy for firm level ambiguity, and an indicator variable set equal to one if the US house of representatives and the US senate are controlled by the same political party as that

[26] These are all restrictions on or specifications of the data that have been used in recent studies of leverage.

of the president, and zero otherwise, as a proxy for economy wide ambiguity.[27,28] These instruments are chosen because they are plausibly exogenous to the firm. In the case of the Control variable, this is the result of popular elections. The Defendant variable is likely to be the result of unintended consequences of their past actions.

Both variables are expected to be directly related to the level of ambiguity firms face. The variable Control reflects the level of policy uncertainty—uncertainty regarding the "rules of the game" for economic activity for a specific period (Baker et al., 2016). The idea being that a unified government is more likely to be able to enact significant policy changes, increasing the level of policy uncertainty. The Defendant variable is a reflection of the level of uncertainty regarding the firm's cash flow and potentially the nature of restrictions on its operations in the near future. The first stage regression tests, shown in Table 5, examine this more completely. Untabulated correlations indicate that the correlation between the Control variable and the first difference in ambiguity (unlevered ambiguity) is 19.8% (19.7%). The estimated correlations between the Defendant variable and the first difference of ambiguity (unlevered ambiguity) is 2.4% (2.0%).

The primary difficulty in identifying instruments for ambiguity is that good instruments for ambiguity may also affect the level of risk faced by a firm. While both of the chosen instruments have significant correlations with ambiguity or unlevered ambiguity, their correlations with the relevant measure of risk in the capital structure decision, cash flow volatility, is lower by at least an order of magnitude.[29] A final concern with *any* instrument in an empirical examination of capital structure is unobserved heterogeneity. Empirical results in Lemmon et al. (2008) indicate that our empirical models of leverage choice suffer from a significant omitted variables problem. Identifying whether our chosen instruments also influence leverage via one of the omitted variables (see Angrist and Pischke (2008) and Atanasov and Black (2016)) is not something that can be examined. We rely on the use of first differences to control for the omitted variables and the argument that the chosen instruments are most likely to influence leverage through the channels of ambiguity, and any influence of the instruments have on leverage via the risk channel is negligible.

The coefficient estimates from the first stage regression using these instruments are used to estimate

[27] The Defendant variable is constructed from the data provided in AuditAnalytics database. The Controlled house of representatives indicator variable is constructed from the data provided at
https://en.wikipedia.org/wiki/Divided_government_in_the_United_States.

[28] The use of variables measured in levels as instruments for the first difference of ambiguity is actually a benefit of using first differences model to control for the unobserved heterogeneity. Doing so implies that the exogeneity required for the instrument is only contemporaneous exogeneity rather than strict exogeneity as would be required in a fixed effects 2-Stage least squares estimation (e.g.,, Wooldridge, 2010).

[29] The estimated correlation between Control and cash flow volatility is 0.5% and the correlation between Defendant and cash flow volatility is 0.1%.

fitted values for the change in ambiguity and unlevered ambiguity. The fitted values for ambiguity are then used in the second stage regression to examine the impact on the change in leverage of plausibly exogenous changes in ambiguity. As before, panel A of Table 5 presents the findings related to book leverage and panel B the findings related to market leverage. In each panel, the first and the third columns present the findings from the first stage regression estimating the first difference of ambiguity or unlevered ambiguity respectively. The second and fourth columns present the findings from the second stage regression tests. As in the other empirical models, the estimates are consistent with the theoretical prediction of our model.

In the first stage regressions, the coefficient estimates on the instruments for ambiguity and unlevered ambiguity are consistently positive and highly significant for both book and market leverage. The coefficient estimates indicate that our measure of ambiguity is indeed higher when firms are named as defendants in a greater number of lawsuits and when there is a greater level of policy uncertainty. The level of significance of these variables and the reported F-statistics for the first stage regression suggest the model does not suffer from a weak instruments problem.

In each second stage regression test, the coefficient estimates for the fitted values of the first difference in ambiguity or unlevered ambiguity indicate a positive and highly significant relation with the first difference in both book and market leverage. The F-statistics are highly significant for both second stage regression specifications. These estimates again demonstrate that our measures of ambiguity are positively related to firms' subsequent leverage choices. Empirically, this aspect of uncertainty has a much more consistent relationship with leverage than does the more commonly examined component, risk.

[Table 5]

A concern that may be raised is whether the relation between ambiguity and leverage is driven by the behavior of firms with very low leverage. To address this concern, Table 6 presents the results of the 2-stage least squares regressions on subsamples of firms with above and below median leverage. The findings, for both groups of firms, are broadly consistent with those reported in Table 5, indicating that this is not a material concern.

For the below median leverage samples, the results for book and market leverage are largely consistent with the results reported in Table 5. For the above median leverage samples, however, the results are somewhat weaker. Namely, for both book and market leverage, the first stage regression test indicates that the instrument "Defendant" is not significantly related to the first difference in either ambiguity or unlevered ambiguity. The indicator variable for the divided government, "Control", has a highly significant relationship with both ambiguity and unlevered ambiguity. The F-statistic for the first stage regression also remains highly significant. For the below and above median book leverage sample, the fitted values of ambiguity and unlevered ambiguity have coefficient estimates that are positive and significantly related to leverage. The F-statistics indicate, however, that the second stage regressions for the below median subsample lose some significance when using book leverage. For market leverage, the second stage regression models remain highly significant.

[Table 6]

4.3 Robustness

We next turn to two examinations of the robustness of our analysis regarding ambiguity. The first is an investigation of the economic significance of ambiguity in explaining capital structure decisions. Table 7 presents measures of the economic significance of the different independent variables we employ. The table is constructed by multiplying the standard deviation of each explanatory variable, shown in the second column, by its coefficient estimate from Table 4 (Panels A and B), with the product of these two quantities displayed in the third column under the heading "Significance." The table reveals three interesting patterns. First, comparing the two components of uncertainty, ambiguity (or unlevered ambiguity) have greater levels of economic significance than does cash flow volatility for explaining leverage choice. Second, the measure of ambiguity displays a greater level of economic significance in explaining leverage than does the measure of unlevered ambiguity. Finally, of the standard control variables, only asset tangibility has a roughly equivalent level of economic significance for explaining leverage as does ambiguity (or unlevered ambiguity). Both ambiguity and unlevered ambiguity display greater economic significance than all of the other common control variables. Overall, the ambiguity faced by the firm appears to have an important impact on its leverage decision.

[Table 7]

Table 8 reports robustness tests using alternative proxies for ambiguity that have been employed in the literature. We test each of the following four factors as a substitute for our ambiguity measure and as a factor alongside our measure. The variance of the mean (Var Mean) is the variance of daily mean returns (computed from 5-minute returns) over a month and averaged over the year. The variance of the variance (Var Var) is the variance of daily variance of returns (computed from 5-minute returns) over a month and averaged over the year. Bid-ask spread is the annual average of the effective bid-ask spread for the firm's equity. Disagreement or dispersion of analysts' forecasts is the variance among analyst forecasts of the future stock price.

We run our regression tests using the first differences of book and market leverage, as explained by year lagged differences in the ambiguity proxies. All regressions include an intercept and changes in the standard control variables that are included in Table 4. Panel A presents the results using book leverage and panel B presents results using market leverage. When the alternative proxies for ambiguity substitute for our measure of ambiguity in the regressions explaining book leverage only the measure of the bid-ask spread is significantly related to leverage. However, the estimated coefficient on the bid-ask spread measure is negative rather than positive. Only the disagreement of analysts' forecasts is positively related to leverage, however that coefficient is insignificant. When the alternative proxies for ambiguity are substituted for our measure and used to explain market leverage (columns 2 - 5 of panel B), all of the estimated coefficients are negative and all but the coefficient estimate for the disagreement of analysts' forecasts are significant.

The final four columns in Panels A and B in Table 8 include both our measure of ambiguity and each of the previously used proxies in regressions that also include risk and the other control variables. Importantly, our measure of ambiguity remains positive and statistically significant in all the regressions for both book and market leverage. The other proxies for ambiguity are all have either significantly negative coefficient estimates or insignificant coefficient estimates. The variance of mean is significantly negatively related to both book and market leverage when included alongside our measure of ambiguity. Since the variance of the mean is often considered a proxy for time varying probability distributions, this finding rules out the possibility that our measure is driven solely by a time varying mean. Similarly, the variance of variance is negatively related to book leverage and significantly negatively related to market leverage when our measure of ambiguity is included in the regression test. For book and market leverage, when our measure of ambiguity and the variance of the variance are included in the regression together, their estimated coefficients are not much different from the estimates obtained when these variables are examined separately. The bid-ask spread has a significantly negative relation to both book and market leverage when included with our measure. Finally, the disagreement among analysts is insignificantly positively related to book leverage and insignificantly negatively related to market leverage when included with our measure.

These findings indicate that our measure of ambiguity and the various proxies for ambiguity capture distinctly different aspects of uncertainty. Overall, the results presented in Table 8 confirm that our measure of ambiguity is not simply a proxy for previously used measures. We also conclude that the previously used proxies for ambiguity, while plausibly reflecting some aspects of ambiguity, do not seem to capture the salient features of this aspect of uncertainty in the context of the capital structure decision. The main reason might be that all the aforementioned alternative measures are outcome-dependent and therefore risk-dependent, while our measure is outcome-independent.

[Table 8]

5 Conclusion

Uncertainty plays a role in the capital structure decision as in many other financial decisions. Until recently, most studies examined uncertainty only by considering risk. However, ambiguity, or Knightian uncertainty, represents a separate and distinct aspect of uncertainty. We contribute to a growing literature by showing that both ambiguity and risk play important roles in capital structure choice.

We present a model that predicts a positive relation between ambiguity and leverage, and test this hypothesis using pooled time series cross-sectional data covering nearly 48,000 firm-year observations from more than 4,200 individual firms. Consistent with the model's prediction, we find a positive association between ambiguity and leverage. The coefficient estimates suggest that the ambiguity facing the firm has a strong, economically meaningful impact on the capital structure decision. Robustness tests demonstrate that other proxies for ambiguity that have been proposed are not meaningful explanatory variables in the context of this decision.

Our results are consistent with other recent papers analyzing variables affected by financial uncertainly, including the pricing of credit default swaps and the timing of the exercise of executive stock options. Together, these studies suggest that the role of uncertainty in financial decisions is richer and more nuanced than previously believed, and that further investigation of ambiguity have the promise of yielding additional insights that may improve our understanding of basic financial decision-making.

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A Appendix

A.1 Firm size and ambiguity

This appendix seeks to provide further evidence regarding the relation between our measure of ambiguity and firm size (or, similarly, firm age). The discussion in Subsection 3.4 implies that the relation between ambiguity and firm size is derived from the fundamental relation between risk and ambiguity, as both of these aspects of uncertainty depend upon the nature of the distributions within the set of possible distributions. While one may expect that riskier firms have a broader set of possible distributions that could determine future outcomes, these distributions are less precise (have greater variance of returns). Thus, a commonly articulated intuition of a positive correlation between risk and ambiguity must be refined.

Panel (a) of Figure 3 depicts the correlation between risk (variance of equity return) and ambiguity for "buckets" of firms of different risk levels. It shows that there is a distinctly different relation between risk and ambiguity for different levels of risk. For firms with relatively high risk (relatively imprecise returns distributions) the common intuition holds true: there is a positive correlation between risk and ambiguity. In contrast, for firms with relatively low risk, the correlation between risk and ambiguity is negative. The reason behind this contrast is that, within the set of relatively high risk firms, when the distributions display greater risk the set of possible distributions is also more dispersed, implying higher ambiguity. Within the set of relatively low risk firms, the lower is the risk, the distributions within the set of possible distributions have greater precision. The greater is the precision of the distributions, the wider the variety of possible probabilities (across the possible distributions) associated with each possible outcome, implying higher variance of the probabilities (higher ambiguity). Panel (b) of Figure 3 depicts the correlation between ambiguity and risk for "buckets" of firms of different sizes. The negative correlation between firm size (or age) and risk shown in Table 2 implies that Panel (b) is essentially a mirror image of Panel (a). Small firms have relatively high risk, which is positively correlated with ambiguity. Large firms have relatively low risk, which is negatively correlated with ambiguity.

[Figure 3]

A second possible cause for the positive relation between the level of ambiguity and firm size is the difference in the nature of available growth opportunities across firms of different sizes. Larger, older, firms, have sets of possible distributions that display greater precision and so higher ambiguity. These same firms typically do not have many organic growth opportunities (Figure 4), those opportunities that exist are likely to be typified by ambiguous prospects (entering new markets or buying growth via mergers). Thus the level of ambiguity for large (old) firms should vary directly with the availability of growth opportunities. Small or young firms have distributions with less precision and so will have lower ambiguity. Smaller and younger firms also tend to have organic growth opportunities (expansion of existing activities) whose characteristics are therefore similar to those of the firm's assets in place. Therefore, for small firms, the level of ambiguity should not be related to the availability of growth opportunities. Panel (a) of Figure 5 depicts the degree of ambiguity across firms within different size bins, sorting firms within each bin by growth opportunities available to the firm (measured by firms' market to book ratio). The figure displays exactly the expected relation between ambiguity and growth opportunities when firms are separated by size. For completeness, Panel (b) of Figure 5 depicts the degree of risk.

[Figure 4]

[Figure 5]

Two things are demonstrated by Panel (a) of Figure 5. First, on average, ambiguity is indeed larger for large firms than it is for small firms. For small firms, where return variance is relatively high, the variance of probabilities across the sets of possible distributions is lower. Second, for the very small and small firms the level of ambiguity is not meaningfully related to the extent of the firm's growth opportunities. Again, the growth opportunities of small firms are generally very similar to the firm's assets in place. These two elements together can explain the low ambiguity and the lack of correlation between ambiguity and growth opportunities in small firms. Panel (a) of Figure 5 also shows that for the large and very large firms the level of ambiguity increases sharply as growth opportunities increase. The reason may be that large firms have relatively limited organic growth opportunities, therefore these opportunities are typified by highly ambiguous prospects.

A.2 Proofs

Proof of Theorem 1.

Substituting the budget constraints into the objective function in Equation (5) and solving the maximization problem, differentiation with respect to θ conditional on a given x (state of nature), provides

$$q(x)\partial_0 \mathrm{U} \quad = \quad \mathrm{E}\left[\varphi\left(x\right)\right]\left(1+\frac{\Upsilon''\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}{\Upsilon'\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}\mathrm{Var}\left[\varphi\left(x\right)\right]\right)\partial_x \mathrm{U}.$$

Organizing terms completes the proof. Since markets are complete and the law of one price holds, the payoff pricing functional assigns a unique price to each state contingent claim. ■

Proof of Theorem 2.

Obtained from the same arguments in Proposition 1 of Modigliani and Miller (1958). ■

Proof of Theorem 3.

The first order condition, obtained by differentiating the firm value in Equation (11) with respect to its leverage, is

$$\frac{\partial V(\cdot)}{\partial F} = \frac{1}{1+r_f}\left(\begin{array}{l} -\int_F^{\infty}\pi^*(x)\,(1-\tau)\,dx \\ +\pi^*(F)F\,(1-\alpha)+\int_F^{\infty}\pi^*(x)dx-\pi^*(F)F \end{array}\right) = 0.$$

Rearranging terms provides

$$F = \frac{\tau}{\pi^*(F)\alpha}\int_F^{\infty}\pi^*(x)dx.$$

■

Proof of Proposition 1.

Consider a payoff $F \leq x$. Suppose that risk increases such that instead of x the payoff is $x+\Delta$ or $x-\Delta$, with equal probabilities, i.e., $x \pm \Delta$ is mean-preserving spread of x. If $\Delta \leq x - F$ this will not affect to value of the equity. If $\Delta > x - F$, then $\frac{1}{2}(x - F + \Delta) > x - F$. This holds true for any $F \leq x$, Thus, by Equation (10), the value of the equity increases in risk. ■

Proof of Proposition 2.

By Equation (7), for every outcome $\pi^*(x)$ decreases in ambiguity. Thus, by Equation (10), S_0 decreases in ambiguity. ■

Proof of Proposition 3.

Substitute the explicit expression of $\pi^*(\cdot)$ into the optimal leverage in Equation (12) to obtain

$$F = \frac{\tau}{\alpha q(F)}\int_F^{\infty}q(x)dx. \tag{18}$$

The first order condition (FOC) can be defined to be the implicit function

$$G\left(F, \mho^2, \mathcal{R}\right) = \tau\int_F^{\infty}q(x)dx - \alpha q(F)F, \tag{19}$$

where $\mathcal{R}$ stands for the level of risk. By the second-order condition, at a maximum

$$\frac{\partial G}{\partial F} < 0.$$

Suppose that for any event with an outcome x the outcome is instead $x \times t$, where $0 < t$. The greater it t the greater is the risk and so is the expected outcome. Differentiating the implicit function in Equation (19) with respect to t, provides

$$\frac{\partial G}{\partial t} = \tau \int_F^\infty \mathrm{E}[\varphi(x)] \left(1 + \frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(x)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(x)])} \mathrm{Var}[\varphi(x)]\right) \frac{\partial_{xx}\mathrm{U}}{\partial_0 \mathrm{U}} x dx.$$

By risk aversion, $\partial_{xx}\mathrm{U} < 0$; Thus, $\frac{\partial G}{\partial t} \leq 0$. By the second-order condition, $\frac{\partial G}{\partial F} < 0$. By the implicit function theorem,

$$\frac{\partial G}{\partial F} dF + \frac{\partial G}{\partial t} dt = 0.$$

Therefore,

$$\frac{dF}{dt} = -\frac{\frac{\partial G}{\partial t}}{\frac{\partial G}{\partial F}} \leq 0.$$

■

Proof of Proposition 4.

Suppose that the ambiguity increases such that the variance $\mathrm{Var}[\varphi(x)] \times t$ of the probability of each x increases, where $t > 0$. Differentiating the implicit function in Equation (19) with respect to t, provides

$$\frac{\partial G}{\partial t} = \tau \int_F^\infty \mathrm{E}[\varphi(x)] \frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(x)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(x)])} \mathrm{Var}[\varphi(x)] \frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}} dx - \alpha \mathrm{E}[\varphi(F)] \frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(x)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(x)])} \mathrm{Var}[\varphi(F)] \frac{\partial_F \mathrm{U}}{\partial_0 \mathrm{U}} F.$$

Since the FOC, defined by Equation (19), is equal to zero,

$$\frac{\partial G}{\partial t} = -\tau \int_F^\infty \mathrm{E}[\varphi(x)] \frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}} dx + \alpha \mathrm{E}[\varphi(F)] \frac{\partial_F \mathrm{U}}{\partial_0 \mathrm{U}} F.$$

Thus, $\frac{\partial G}{\partial t} \geq 0$ when

$$F \geq \frac{\tau}{\alpha} \frac{1}{\mathrm{E}[\varphi(F)] \frac{\partial_F \mathrm{U}}{\partial_0 \mathrm{U}}} \int_F^\infty \mathrm{E}[\varphi(x)] \frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}} dx. \tag{20}$$

The right hand side of this inequality is the optimal F assuming there is no ambiguity (alternatively, no aversion to ambiguity). Since $\left(\frac{\Upsilon''}{\Upsilon'}\right)' \geq 0$, $0 \geq \frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(F)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(F)])} \geq \frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(x)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(x)])}$ for any $x \geq F$. Therefore, this inequality holds when ambiguity is present. By the implicit function theorem,

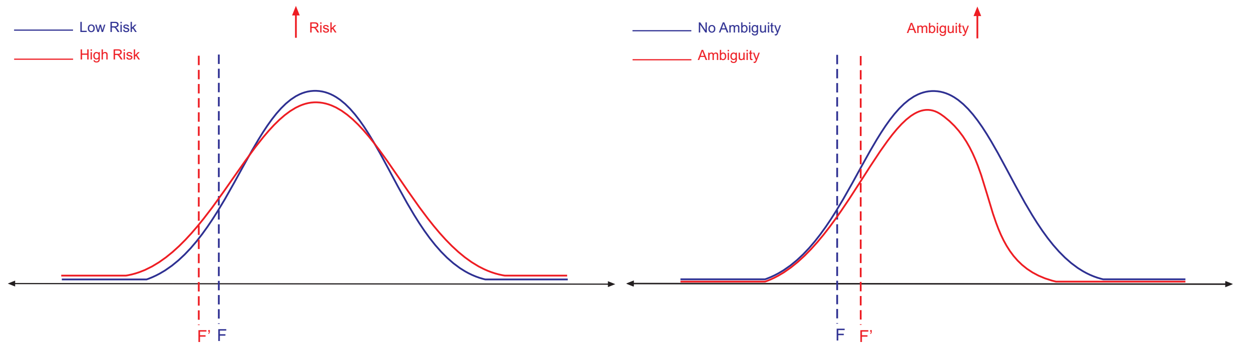
$$\frac{\partial G}{\partial F} dF + \frac{\partial G}{\partial t} dt = 0.$$

Therefore, since $\frac{\partial G}{\partial F} < 0$,

$$\frac{dF}{dt} = -\frac{\frac{\partial G}{\partial t}}{\frac{\partial G}{\partial F}} \geq 0.$$

■

A.3 Tables and figures



(a) Increasing Risk

(b) Increasing Ambiguity

Figure 1: **Optimal leverage**

This figure depicts the effect of an increase in risk (left panel) and an increase in ambiguity (right panel) on optimal leverage. Each panel represents a stylized perceived probability distribution in which the x-axis illustrates the range of the firm's possible future values. The point F is the optimal choice for the firm's leverage, at which the expected marginal benefit of interest tax shields just equals the expected marginal cost of bankruptcy. In the left panel, the blue series shows a lower-risk perceived probability distribution, which is assumed to shift to the red series, representing a higher risk. The optimal choice of leverage then moves to the left on the x-axis. In the right panel, an increase in ambiguity is assumed to occur, resulting in lower perceived probabilities of expected outcomes, as shown by the change from the blue to the red probability distribution. In this case the optimal choice of leverage moves to the right.

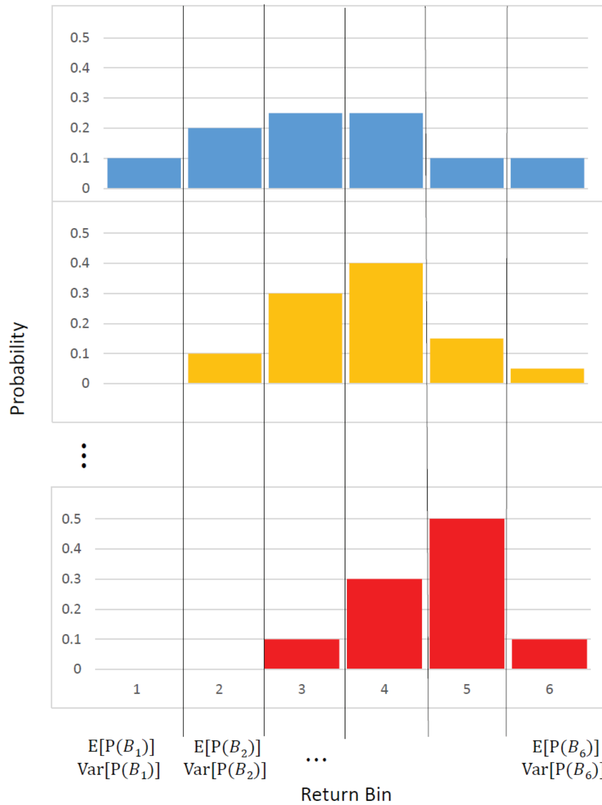
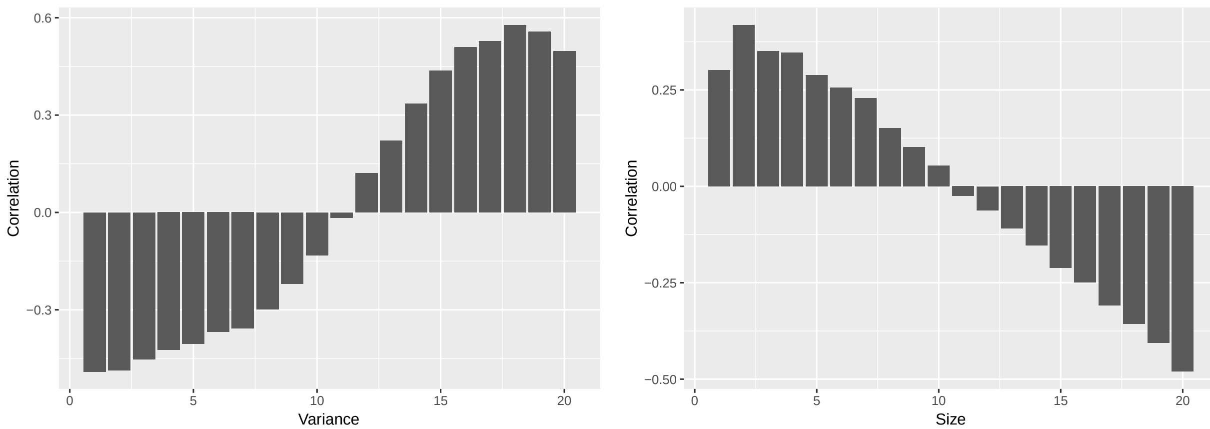


Figure 2: **Ambiguity measurement**

This figure illustrates the computation of the ambiguity measure, which is derived for each firm-month based on intraday stock-returns sampled at a five-minute frequency from 9:30 to 16:00. Thus, we obtain up to 22 daily histograms of up to 78 intraday returns in each month. We discretize the daily return distributions into n bins of equal size $B_j = (r_j, r_{j-1}]$ across histograms. The height of the histogram for a particular bin is computed as the fraction of daily intraday returns observed in that bucket, and thus represents the probability of that particular bin outcome. We compute the expected probability of being in a particular bin across the daily return distributions, $\mathrm{E}\left[\mathrm{P}_i\left(B_j\right)\right]$, as well as the variance of these probabilities, $\mathrm{Var}\left[\mathrm{P}_i\left(B_j\right)\right]$. Ambiguity is then computed as $\mho^2\left[r_i\right] \equiv 1/\sqrt{w\left(1-w\right)}\sum_{j=1}^{n} \mathrm{E}\left[\mathrm{P}_i\left(B_j\right)\right] \mathrm{Var}\left[\mathrm{P}_i\left(B_j\right)\right]$, where we scale the weighted-average volatilities of probabilities to the bins' size $w = r_{i,j} - r_{i,j-1}$.



(a) By Risk

(b) By Size

Figure 3: **Correlation between ambiguity and risk**

This figure depicts the correlation between the monthly ambiguity, measured by $\mho^2$, and the monthly risk, measured by the equity return variance. Panel (a) depicts this correlation sorted by the firms' risk level. Panel (b) depicts this correlation sorted by the firms' size. The correlation between ambiguity and risk is computed for each firm separably, and sorted to brackets based on the average firm size over the data sample.

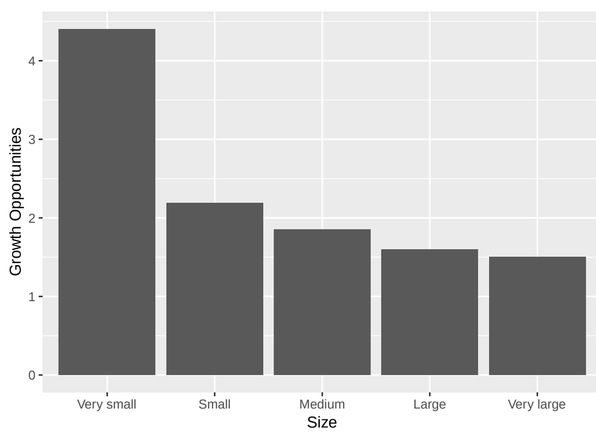
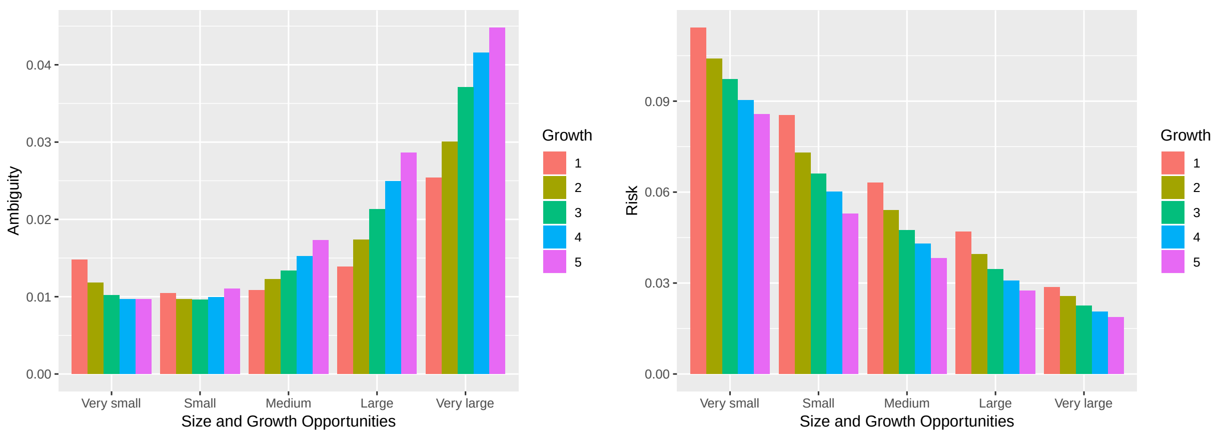


Figure 4: **Growth opportunities by size**

This figure depicts firm growth opportunities, measured by the within quintile average market to book ratio, for firms sorted by size.



(a) Ambiguity

(b) Risk

Figure 5: **Ambiguity and risk by size and growth opportunities**

Panel (a) depicts the monthly degree of ambiguity, measured by $\mho^2$, and sorted by the firms' size and then by firm's growth opportunities, measured by the market to book ratio. Panel (b) depicts the monthly degree of risk, measured by equity return variance, and sorted by the firms' size and then by firm's growth opportunities.

Table 1: **Descriptive statistics**

The full sample includes 47,915 annual observations associated with 4,242 individual firms between 1993 and 2017, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Sale is the log of sales normalized to the annual gross domestic product (GDP) level. Profitability is operating income before depreciation divided by book assets. Tangibility is the plant property divided by book assets. The market to book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales. Median leverage is the median of the annual leverage in the industry.

Variable	Mean	Std Dev	Minimum	Median	Maximum	N
Book Leverage	0.199	0.190	0.000	0.168	0.875	47,915
Market Leverage	0.177	0.198	0.000	0.113	0.886	47,915
Ambiguity	0.019	0.019	0.001	0.012	0.125	47,915
Ambiguity Unlev.	0.028	0.032	0.001	0.016	0.315	47,915
Risk	0.213	0.162	0.033	0.153	1.115	47,915
Cash Flow Vol.	0.032	0.230	0.00000	0.0003	5.432	47,915
Median Book Lev.	0.171	0.149	0.000	0.142	0.869	47,915
Median Book Lev.	0.153	0.159	0.000	0.103	0.966	47,915
Profitability	0.076	0.199	-1.512	0.116	0.485	47,915
Tangibility	0.248	0.219	0.001	0.178	0.920	47,915
Market to Book	1.794	1.473	0.000	1.324	12.181	47,915
R&D	0.023	0.110	0	0.001	2	47,915
Sales	1.834	1.348	0.000	1.632	5.729	47,915
Tax Rate	0.152	0.153	0.000	0.046	0.395	47,915
Firm Age	18.047	12.573	2	14	60	47,915

Table 2: **Correlation matrix**

The full sample includes 47,915 annual observations associated with 4,242 individual firms between 1993 and 2017, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by total book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Sale is the log of sales normalized to the annual gross domestic product (GDP) level. Profitability is operating income before depreciation divided by book assets. Tangibility is plant property and equipment divided by book assets. The market to book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales.

	Book Leverage	Market Leverage	Ambiguity	Ambiguity (Unlev)	Risk	Cash Flow Vol.	Profitability	Tangibility	Market to Book	R&D	Sales	Tax Rate	Firm Age
Book Leverage	1												
Market Leverage	0.814	1											
Ambiguity	0.002	-0.022	1										
Ambiguity Unlev.	0.082	0.033	0.906	1									
Risk	0.066	0.142	-0.351	-0.422	1								
Cash Flow Vol.	0.051	0.060	0.051	0.064	-0.108	1							
Profitability	-0.143	-0.241	0.002	-0.008	-0.104	-0.036	1						
Tangibility	0.086	0.089	-0.089	-0.106	0.236	-0.022	-0.044	1					
Market to Book	-0.098	-0.424	0.010	0.011	-0.103	-0.044	0.305	-0.004	1				
R&D	0.038	0.054	0.126	0.142	-0.114	0.074	-0.354	-0.033	-0.077	1			
Sales	0.067	0.073	0.156	0.187	-0.324	0.215	0.283	-0.052	-0.061	-0.170	1		
Tax Rate	-0.055	-0.077	-0.035	-0.038	-0.020	-0.055	0.244	-0.017	0.075	-0.173	0.097	1	
Firm Age	0.085	0.094	0.270	0.324	-0.485	0.180	-0.104	-0.230	-0.140	0.294	0.199	-0.061	1

Table 3: **Level regression estimates of capital structure**

Linear model regression estimates of the level of annual leverage ratio explained by one-year lagged variables. The full sample includes 47,915 annual observations associated with 4,242 individual firms between 1993 and 2017, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Sale is the log of sales normalized by the annual gross domestic product (GDP) level. Profitability is operating income before depreciation divided by book assets. Tangibility is plant property and equipment divided by book assets. The market to book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is the R&D expenses relative to sales. Standard errors, clustered by firm and robust to heteroscedasticity, appear in parentheses below each coefficient estimate.

Panel A: Book leverage

Ambiguity	1.690^{***}		1.660^{***}	0.665^{***}				
	(0.094)		(0.094)	(0.086)				
Ambiguity Unlev.					2.053^{***}		2.055^{***}	1.355^{***}
					(0.056)		(0.056)	(0.059)
Cash Flow Vol.		0.034^{***}	0.016^{**}	-0.017^{***}		0.034^{***}	-0.002	-0.014^{***}
		(0.008)	(0.008)	(0.005)		(0.008)	(0.007)	(0.004)
Median Book Lev.				0.594^{***}				0.553^{***}
				(0.014)				(0.014)
Profitability				-0.047^{***}				-0.050^{***}
				(0.009)				(0.009)
Tangibility				0.090^{***}				0.099^{***}
				(0.010)				(0.010)
Market to Book				-0.008^{***}				-0.010^{***}
				(0.001)				(0.001)
R&D				-0.00000				-0.00001
				(0.00001)				(0.00000)
Sales				0.018^{***}				0.007^{***}
				(0.002)				(0.002)
Tax Rate				-0.092^{***}				-0.077^{***}
				(0.010)				(0.009)
Firm Age				-0.001^{***}				-0.001^{***}
				(0.0002)				(0.0001)
Constant	0.167^{***}	0.198^{***}	0.167^{***}	0.077^{***}	0.143^{***}	0.198^{***}	0.143^{***}	0.084^{***}
	(0.003)	(0.003)	(0.003)	(0.005)	(0.003)	(0.003)	(0.003)	(0.005)
Observations	42,561	42,561	42,561	42,561	42,561	42,561	42,561	42,561
R^2	0.030	0.002	0.031	0.344	0.117	0.002	0.117	0.374

Panel B: Market leverage

Ambiguity	0.426***		0.372***	−0.294***				
	(0.086)		(0.086)	(0.078)				
Ambiguity Unlev.					1.080***		1.064***	0.467***
					(0.061)		(0.061)	(0.050)
Cash Flow Vol.		0.033***	0.029***	−0.014***		0.033***	0.014*	−0.012***
		(0.009)	(0.009)	(0.005)		(0.009)	(0.009)	(0.005)
Median Market Lev.				0.537***				0.540***
				(0.014)				(0.014)
Profitability				−0.054***				−0.057***
				(0.007)				(0.007)
Tangibility				0.095***				0.097***
				(0.010)				(0.010)
Market to Book				−0.027***				−0.028***
				(0.001)				(0.001)
R&D				−0.00001***				−0.00001***
				(0.00000)				(0.00000)
Sales				0.023***				0.015***
				(0.002)				(0.002)
Tax Rate				−0.096***				−0.087***
				(0.009)				(0.009)
Firm Age				−0.001***				−0.001***
				(0.0002)				(0.0002)
Constant	0.171***	0.178***	0.171***	0.119***	0.149***	0.178***	0.149***	0.119***
	(0.003)	(0.003)	(0.003)	(0.005)	(0.003)	(0.003)	(0.003)	(0.005)
Observations	42,561	42,561	42,561	42,561	42,561	42,561	42,561	42,561
R^2	0.002	0.002	0.003	0.393	0.029	0.002	0.030	0.396

Note: *p<0.1; **p<0.05; ***p<0.01

Table 4: **Changes regression estimates of capital structure**

Linear model regression estimates of the changes in annual leverage ratio explained by one-year lagged changes in the variables. The full sample includes 47,915 annual observations associated with 4,242 individual firms between 1993 and 2017, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Sale is the log of sales normalized by the annual gross domestic product (GDP) level. Profitability is operating income before depreciation divided by book assets. Tangibility is plant property and equipment divided by book assets. The market to book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is the R&D expenses relative to sales. Standard errors, clustered by firm and robust to heteroscedasticity, appear in parentheses below each coefficient estimate.

Panel A: Book leverage

Ambiguity	0.153^{***}		0.154^{***}	0.168^{***}				
	(0.036)		(0.036)	(0.036)				
Ambiguity Unlev.					0.047^{**}		0.048^{**}	0.059^{**}
					(0.024)		(0.024)	(0.024)
Cash Flow Vol.		0.001	0.002	0.002		0.001	0.002	0.002
		(0.002)	(0.002)	(0.002)		(0.002)	(0.002)	(0.002)
Median Book Lev.				0.001				0.0001
				(0.007)				(0.007)
Profitability				0.004				0.004
				(0.005)				(0.005)
Tangibility				0.081^{***}				0.080^{***}
				(0.012)				(0.012)
Market to Book				−0.001				−0.001
				(0.0004)				(0.0004)
R&D				0.000				0.000
				(0.000)				(0.000)
Sales				0.003				0.003
				(0.003)				(0.003)
Tax Rate				−0.002				−0.002
				(0.004)				(0.004)
Constant	0.005^{***}	0.005^{***}	0.005^{***}	0.005^{***}	0.005^{***}	0.005^{***}	0.005^{***}	0.005^{***}
	(0.0003)	(0.0003)	(0.0003)	(0.0003)	(0.0003)	(0.0003)	(0.0003)	(0.0003)
Observations	35,444	35,444	35,444	35,444	35,444	35,444	35,444	35,444
R^2	0.0005	0.00001	0.0005	0.003	0.0001	0.00001	0.0001	0.002

Panel B: Market leverage

Ambiguity	0.458^{***}		0.456^{***}	0.430^{***}				
	(0.045)		(0.045)	(0.044)				
Ambiguity Unlev.					0.369^{***}		0.368^{***}	0.346^{***}
					(0.032)		(0.032)	(0.032)
Cash Flow Vol.		-0.006^{**}	-0.004	-0.005^{*}		-0.006^{**}	-0.004	-0.005^{*}
		(0.003)	(0.003)	(0.003)		(0.003)	(0.003)	(0.003)
Median Market Lev.				-0.011				-0.009
				(0.007)				(0.007)
Profitability				-0.005				-0.005
				(0.005)				(0.005)
Tangibility				0.051^{***}				0.053^{***}
				(0.014)				(0.014)
Market to Book				0.001^{***}				0.001^{***}
				(0.0003)				(0.0003)
R&D				0.000				0.000
				(0.000)				(0.000)
Sales				0.042^{***}				0.041^{***}
				(0.004)				(0.004)
Tax Rate				-0.005				-0.005
				(0.005)				(0.005)
Constant	0.006^{***}	0.007^{***}	0.006^{***}	0.005^{***}	0.006^{***}	0.007^{***}	0.006^{***}	0.005^{***}
	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)
Observations	35,444	35,444	35,444	35,444	35,444	35,444	35,444	35,444
R^2	0.003	0.0001	0.003	0.008	0.004	0.0001	0.004	0.009

Note: *p<0.1; **p<0.05; ***p<0.01

Table 5: **First Difference 2-Stage Least Squares regression estimates of capital structure**

The full sample includes 47,915 annual observations associated with 4,242 individual firms between 1993 and 2017, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Sale is the log of sales normalized by the annual gross domestic product (GDP) level. Profitability is operating income before depreciation divided by book assets. Tangibility is plant property and equipment divided by book assets. The market to book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is the R&D expenses relative to sales. The instrumental variable Defendant is the number of lawsuits in which the firm is named as a defendant in each year, taken as a proxy for the firm level ambiguity. The instrumental variable Control is an indicator variable set equal to one if the US house of representatives and the US senate are controlled by different political parties, and zero otherwise, taken as a proxy for economy wide ambiguity. Standard errors, clustered by firm and robust to heteroscedasticity, appear in parentheses below each coefficient estimate.

Panel A: Book leverage

Ambiguity(fit)		1.073^{***}		
		(0.224)		
Ambiguity Unlev.(fit)				0.804^{***}
				(0.168)
Control	0.004^{***}		0.005^{***}	
	(0.0001)		(0.0002)	
Defendant	0.0002^{***}		0.0002^{**}	
	(0.0001)		(0.0001)	
Cash Flow Vol.	-0.004^{***}	0.006^{**}	-0.006^{***}	0.006^{**}
	(0.001)	(0.003)	(0.001)	(0.003)
Median Book Lev.	-0.006^{***}	-0.006	-0.005^{***}	-0.008
	(0.001)	(0.007)	(0.002)	(0.007)
Profitability	-0.002^{***}	0.008	-0.003^{***}	0.008
	(0.001)	(0.006)	(0.001)	(0.006)
Tangibility	-0.007^{***}	0.094^{***}	-0.014^{***}	0.098^{***}
	(0.001)	(0.014)	(0.002)	(0.014)
Market to Book	0.0004^{***}	-0.001^{**}	0.001^{***}	-0.001^{**}
	(0.0001)	(0.0004)	(0.0001)	(0.0004)
R&D	0.000^{*}	0.000	0.000^{**}	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
Sales	-0.0002	0.001	0.002^{**}	-0.001
	(0.0004)	(0.003)	(0.001)	(0.003)
Tax Rate	0.0004	-0.002	0.0001	-0.002
	(0.001)	(0.004)	(0.001)	(0.004)
Observations	35,444	35,444	35,444	35,444
F-statistics	508.6	22.9	419.4	22.9

Panel B: Market leverage

Ambiguity(fit)		4.459^{***}		
		(0.308)		
Ambiguity Unlev.(fit)				3.362^{***}
				(0.232)
Control	0.004^{***}		0.005^{***}	
	(0.0001)		(0.0002)	
Defendant	0.0002^{***}		0.0002^{***}	
	(0.0001)		(0.0001)	
Cash Flow Vol.	-0.004^{***}	0.014^{***}	-0.006^{***}	0.015^{***}
	(0.001)	(0.005)	(0.001)	(0.005)
Median Market Lev.	-0.016^{***}	0.044^{***}	-0.025^{***}	0.055^{***}
	(0.001)	(0.010)	(0.001)	(0.010)
Profitability	-0.003^{***}	0.006	-0.004^{***}	0.008
	(0.001)	(0.006)	(0.001)	(0.006)
Tangibility	-0.005^{***}	0.080^{***}	-0.012^{***}	0.096^{***}
	(0.001)	(0.016)	(0.002)	(0.016)
Market to Book	0.0003^{***}	-0.00005	0.0004^{***}	-0.0001
	(0.00004)	(0.0003)	(0.0001)	(0.0003)
R&D	0.000	0.000	0.000^{**}	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
Sales	0.0003	0.039^{***}	0.002^{***}	0.032^{***}
	(0.0004)	(0.005)	(0.001)	(0.005)
Tax Rate	0.0002	-0.005	-0.0002	-0.003
	(0.001)	(0.006)	(0.001)	(0.006)
Observations	35,444	35,444	35,444	35,444
F-statistics	487.6	210.0	396.0	210.6

Note: *p<0.1; **p<0.05; ***p<0.01

Table 6: **First Difference 2-Stage Least Squares estimates of capital structure in subgroups by median**

The full sample includes 47,915 annual observations associated with 4,242 individual firms between 1993 and 2017, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Sale is the log of sales normalized by the annual gross domestic product (GDP) level. Profitability is operating income before depreciation divided by book assets. Tangibility is plant property and equipment divided by book assets. The market to book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is the R&D expenses relative to sales. The instrumental variable Defendant is the number of lawsuits in which the firm is named as a defendant in each year, taken as a proxy for the firm level ambiguity. The instrumental variable Control is an indicator variable set equal to one if the US house of representatives and the US senate are controlled by different political parties, and zero otherwise, taken as a proxy for economy wide ambiguity. Standard errors, clustered by firm and robust to heteroscedasticity, appear in parentheses below each coefficient estimate.

Panel A: Book leverage

	Below Median				Above Median			
Ambiguity(fit)		0.918^{***}				1.768^{***}		
		(0.242)				(0.352)		
Ambiguity Unlev.(fit)				0.864^{***}				1.117^{***}
				(0.227)				(0.223)
Control	0.003^{***}		0.004^{***}		0.005^{***}		0.008^{***}	
	(0.0002)		(0.0002)		(0.0002)		(0.0004)	
Defendant	0.0002^{**}		0.0003^{**}		0.0001		0.0002	
	(0.0001)		(0.0001)		(0.0001)		(0.0001)	
Cash Flow Vol.	-0.005^{***}	0.003	-0.007^{***}	0.004	-0.004^{***}	0.009^{**}	-0.006^{***}	0.009^{**}
	(0.001)	(0.003)	(0.001)	(0.003)	(0.001)	(0.004)	(0.002)	(0.004)
Median Book Lev.	0.001	-0.015^{*}	0.003^{*}	-0.017^{**}	-0.010^{***}	-0.029^{**}	-0.012^{***}	-0.033^{***}
	(0.001)	(0.008)	(0.002)	(0.008)	(0.002)	(0.012)	(0.003)	(0.012)
Profitability	-0.001^{**}	0.012^{***}	-0.002^{**}	0.012^{***}	-0.003^{*}	0.029	-0.004^{**}	0.029
	(0.001)	(0.004)	(0.001)	(0.004)	(0.001)	(0.018)	(0.002)	(0.018)
Tangibility	-0.003	0.060^{***}	-0.005^{**}	0.061^{***}	-0.011^{***}	0.119^{***}	-0.023^{***}	0.126^{***}
	(0.002)	(0.012)	(0.002)	(0.012)	(0.002)	(0.024)	(0.004)	(0.024)
Market to Book	0.0002^{***}	-0.001^{***}	0.0003^{***}	-0.001^{***}	0.001^{***}	-0.0004	0.002^{***}	-0.001
	(0.00005)	(0.0003)	(0.00005)	(0.0003)	(0.0002)	(0.002)	(0.0003)	(0.002)
R&D	0.000	0.000	0.000	0.000	0.000^{**}	-0.000	0.000^{***}	-0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Sales	-0.002^{***}	-0.008^{**}	-0.001^{**}	-0.008^{**}	0.001	0.00004	0.004^{***}	-0.003
	(0.001)	(0.004)	(0.001)	(0.004)	(0.001)	(0.006)	(0.001)	(0.006)
Tax Rate	0.001^{*}	-0.006	0.001	-0.006	-0.0004	0.004	-0.001	0.004
	(0.001)	(0.004)	(0.001)	(0.004)	(0.001)	(0.007)	(0.001)	(0.007)
Observations	17,722	17,722	17,722	17,722	17,722	17,722	17,722	17,722
F-statistics	224.4	14.4	202.4	14.5	24.2	25.2	243.3	25.2

Panel B: Market leverage

	Below Median				Above Median			
Ambiguity(fit)		1.358^{***}				7.118^{***}		
		(0.204)				(0.537)		
Ambiguity Unlev.(fit)				1.218^{***}				4.636^{***}
				(0.193)				(0.352)
Control	0.004^{***}		0.004^{***}		0.005^{***}		0.007^{***}	
	(0.0002)		(0.0002)		(0.0002)		(0.0003)	
Defendant	0.0003^{***}		0.0004^{***}		0.0001		0.0001	
	(0.0001)		(0.0001)		(0.0001)		(0.0001)	
Cash Flow Vol.	-0.004^{***}	0.006^{*}	-0.005^{***}	0.006^{*}	-0.004^{***}	0.024^{***}	-0.006^{***}	0.023^{***}
	(0.001)	(0.003)	(0.002)	(0.004)	(0.001)	(0.008)	(0.001)	(0.008)
Median Market Lev.	-0.008^{***}	-0.017^{**}	-0.009^{***}	-0.017^{**}	-0.019^{***}	0.084^{***}	-0.031^{***}	0.094^{***}
	(0.001)	(0.007)	(0.002)	(0.007)	(0.001)	(0.016)	(0.002)	(0.017)
Profitability	-0.002^{***}	0.009^{**}	-0.002^{***}	0.009^{**}	-0.006^{***}	0.039^{*}	-0.009^{***}	0.040^{*}
	(0.001)	(0.004)	(0.001)	(0.004)	(0.002)	(0.021)	(0.002)	(0.021)
Tangibility	-0.003	0.011	-0.006^{***}	0.014	-0.008^{***}	0.145^{***}	-0.018^{***}	0.170^{***}
	(0.002)	(0.011)	(0.002)	(0.011)	(0.002)	(0.033)	(0.004)	(0.033)
Market to Book	0.0002^{***}	0.001^{***}	0.0002^{***}	0.001^{***}	0.001^{***}	-0.001	0.003^{***}	-0.003
	(0.00004)	(0.0002)	(0.00004)	(0.0002)	(0.0003)	(0.003)	(0.0004)	(0.003)
R&D	0.000	-0.000	0.000	-0.000	0.000	-0.00001	-0.000	-0.00001
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.00001)	(0.000)	(0.00001)
Sales	-0.001^{*}	0.014^{***}	-0.001	0.014^{***}	0.001^{**}	0.051^{***}	0.005^{***}	0.037^{***}
	(0.001)	(0.004)	(0.001)	(0.004)	(0.001)	(0.008)	(0.001)	(0.009)
Tax Rate	0.001^{**}	0.001	0.001	0.002	-0.0005	-0.002	-0.001	-0.001
	(0.001)	(0.004)	(0.001)	(0.004)	(0.001)	(0.010)	(0.001)	(0.010)
Observations	17,722	17,722	17,722	17,722	17,722	17,722	17,722	17,722
F-statistics	214.3	44.2	178.9	39.9	267.2	175.8	234.4	173.2

Note: *p<0.1; **p<0.05; ***p<0.01

Table 7: **Economic significance of the estimates of capital structure**

Linear model regression estimates of the changes in annual leverage ratio explained by one-year lagged changes in the variables. The full sample includes 47,915 annual observations associated with 4,242 individual firms between 1993 and 2017, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt". Sale is the log of sales normalized by the annual gross domestic product (GDP) level. Profitability is operating income before depreciation divided by book assets. Tangibility is plant property and equipment divided by book assets. The market to book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is the R&D expenses relative to sales. Standard errors, clustered by firm and robust to heteroscedasticity, appear in parentheses below each coefficient estimate.

Panel A: Book leverage changes

	Std Dev.	Coefficient	Significance	Ambiguity Relative to	Unlevered Ambiguity Relative to
Ambiguity	0.010	0.200	15.900	1.000	0.400
Ambiguity Unlev.	0.020	0.100	3.800	2.800	1.000
Cash Flow Vol.	0.100	0.002	0.020	82.400	28.900
Median Book Lev.	0.100	0.001	0.010	204.100	71.600
Profitability	0.100	0.004	0.040	42.300	14.800
Tangibility	0.040	0.100	2.000	2.100	0.700
Market to Book	1.300	-0.001	-0.000	301.500	105.800
R&D	6528.400	0.000	0.000	2993759.000	1050529.000
Sales	0.200	0.003	0.020	56.700	19.900
Tax Rate	0.100	-0.002	-0.020	97.100	34.100

Panel B: Market leverage changes

	Std Dev.	Coefficient	Significance	Ambiguity Relative to	Unlevered Ambiguity Relative to
Ambiguity	0.010	0.400	40.600	1.00	0.800
Ambiguity Unlev.	0.020	0.300	22.500	1.200	1.000
Cash Flow Vol.	0.100	-0.010	-0.040	79.000	63.600
Median Market Lev.	0.100	-0.010	-0.100	39.800	32.000
Profitability	0.100	-0.010	-0.100	78.800	63.500
Tangibility	0.040	0.100	1.300	8.500	6.800
Market to Book	1.300	0.001	0.001	486.000	391.100
R&D	6528.400	0.000	0.000	14212629.000	11438101.000
Sales	0.200	0.040	0.300	10.300	8.300
Tax Rate	0.100	-0.005	-0.040	89.600	72.000

Table 8: **Robustness tests**

Linear model regression estimates of the changes in annual leverage ratio explained by one-year lagged changes in the variables. The full sample includes 47,915 annual observations associated with 4,242 individual firms between 1993 and 2017, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is “debt in current liabilities” plus “long-term debt” divided by book assets. Market leverage ratio is “debt in current liabilities” plus “long-term debt” divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus “debt in current liabilities” plus “long-term debt”. The variance of the mean (Var mean) is the variance of daily mean returns (computed from 5-minute return) over the month. The variance of the variance (Var var) is the variance of daily variance returns (computed from 5-minute return) over the month. Bid-ask spread is the effective bid-ask spread. Analysts disagreement is the variance among analyst forecasts of the stock price. All regressions include an intercept and the following control variables (whose coefficient estimates are unreported). Sale is the log of sales normalized by the annual gross domestic product (GDP) level. Profitability is operating income before depreciation divided by book assets. Tangibility is plant property and equipment divided by book assets. The market to book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is the R&D expenses relative to sales. Standard errors, clustered by firm and robust to heteroscedasticity, appear in parentheses below each coefficient estimate.

Panel A: Book leverage

Ambiguity	0.152^{***}					0.169^{***}	0.162^{***}	0.158^{***}	0.179^{***}
	(0.041)					(0.043)	(0.041)	(0.041)	(0.055)
Cash Flow Vol.	0.003	0.002	0.002	0.002	0.005^{*}	0.003	0.003	0.003	0.005^{**}
	(0.002)	(0.002)	(0.002)	(0.002)	(0.003)	(0.002)	(0.002)	(0.002)	(0.003)
Var mean		-0.294				-0.367^{*}			
		(0.208)				(0.214)			
Var var			-4.513				-6.805		
			(5.768)				(5.885)		
Bid-ask spread				-0.149^{***}				-0.155^{***}	
				(0.040)				(0.041)	
Analysts disagr.					0.001				0.001
					(0.003)				(0.003)
Observations	36,673	36,673	36,673	36,673	21,168	36,673	36,673	36,673	21,168
R^2	0.002	0.002	0.002	0.002	0.004	0.003	0.003	0.003	0.004

Panel B: Market leverage

Ambiguity	0.418***					0.509***	0.474***	0.437***	0.357***
	(0.049)					(0.052)	(0.050)	(0.050)	(0.062)
Cash Flow Vol.	−0.006**	−0.007**	−0.008**	−0.007**	−0.005	−0.005*	−0.006*	−0.005*	−0.003
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
Var mean		−1.636***				−1.870***			
		(0.219)				(0.227)			
Var var			−30.482***				−37.360***		
			(6.546)				(6.856)		
Bid-ask spread				−0.485**				−0.501**	
				(0.233)				(0.238)	
Analysts disagr.					−0.002				−0.002
					(0.003)				(0.003)
Observations	36,673	36,673	36,673	36,673	21,168	36,673	36,673	36,673	21,168
R^2	0.009	0.011	0.008	0.010	0.007	0.014	0.011	0.013	0.009

Note: *p<0.1; **p<0.05; ***p<0.01