

ESG Investing: How to Optimize Impact?

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Abstract

This paper develops a general equilibrium model of a productive economy with negative externalities. Investors are not willing to accept lower returns than their best investment alternatives and entrepreneurs maximize profits. If capital markets are subject to a search friction, an ESG fund can raise assets and improve social welfare despite the selfishness of all agents. The presence of the ESG fund forces companies to partially internalize externalities. We derive the fund's optimal policy in terms of industry allocation and pollution limits imposed to portfolio companies. The fund prioritizes investments in companies where (i) the inefficiency induced by the externality is particularly acute and (ii) the capital search friction is strong. We also show that the ESG fund can take advantage of the supply-chain network: It can amplify its impact by imposing restrictions on the suppliers of the firms where it invests.

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1 Introduction

Negative externalities generated by corporations, such as pollution, are a central theme in current policy debates. The traditional economic prescription to solve such externalities is regulation: Via Pigouvian taxes or tradable pollution permits (“cap-and-trade”), governments can influence the decisions of firms, thereby forcing them to internalize externalities (Weitzman (1974); Cropper and Oates (1992)). Due to political economy constraints, this approach has sometimes delivered disappointing results. Consider the example of carbon emissions: Free-riding among countries, political short-termism, and lobbying frictions, have strongly inhibited the regulatory response to climate change (see e.g. Tirole (2012)).

An alternative channel to curb firms’ behavior is the financing channel: The participation of socially responsible investors to financial markets might relax financial constraint and/or decrease the cost of capital for companies that act responsibly, hence providing incentives to behave better. More and more investors do actually use sustainability criteria in their investment policy: According to the *The Forum for Sustainable and Responsible Investment*, as of year-end 2017, about 25% of U.S. professionally managed assets can be categorized as “socially responsible”. Broadly speaking, one can identify two reasons for an individual to invest via a responsible financial intermediary. First, a *non-consequentialist* view that consists of an intrinsic preference for financing responsible firms regardless on whether this has an impact or not on the level of negative externality in the economy. Second, a *consequentialist* approach that aims at investing via financial intermediaries, whose objective is to have a real impact in the economy by reducing negative externality, regardless on the firms in which funds are actually invests.

This paper embraces the consequentialist view and aims at answering the following question. Consider a responsible financial intermediary whose objective is to have impact, i.e. to improve social welfare by reducing externalities. How should this intermediary choose the composition of its portfolio, and what behavior can it request from the firms it finances? The answer is not obvious for two main reasons. The first one is substitution: Companies that are not compliant with the wishes of responsible investors might simply seek capital elsewhere. The second reason is financial

performance: Most responsible investors insist on generating returns that are competitive with non-responsible alternatives, which restricts their feasible investment strategies.¹

To answer this question, we model a multi-sector competitive economy where the two constraints mentioned here above are taken on board. There is a continuum of atomistic entrepreneurs and investors. Investors invest their capital via financial intermediaries, which we refer to as funds.² Investors can invest via profit-maximizing funds and a responsible fund (the “ESGF”), which cares about aggregate welfare. Entrepreneurs raise capital to produce and they can choose the amount of pollution involved in their production process. Pollution increases production levels at no direct cost to the polluting firms. However, the aggregate level of pollution affects individual welfare negatively. To be conservative, we impose that no investor is willing to accept lower returns than her best investment alternatives. Hence, a responsible fund cannot raise capital if its returns are less than what investors can achieve via other funds. An additional difficulty for the ESGF is that companies can raise capital from non-responsible investors: This substitutability makes it hard for the ESGF to impact companies’ behavior. We introduce a matching friction (à la Duffie et al. (2005)) in capital markets, so that we can parametrize how easy it is for companies to finance themselves without recourse to the ESGF. The optimal policy of the ESGF is defined by its capital allocation across sectors and the pollution requirements it imposes on companies if they decide to accept its capital. We compute the optimal policy of the ESGF, as a function of its assets under management. In equilibrium, the presence of the ESGF increases aggregate welfare but reduces aggregate production and consumption.

We find several results, which have concrete normative implications for the sustainable finance industry. First we show that if the ESGF just defines its strategy as a cross-sector capital allocation, then it has no impact on social welfare. To have an impact, the ESGF must impose some binding pollution caps to the firms it finances. Second, we show that it is optimal for the ESGF to apply a pecking order: It prioritizes investment in sectors where the laissez-faire equilibrium externality

¹This might be due both to preferences and to legal constraints: For instance, in the US, investors subject to the fiduciary duties defined by the Employee Retirement Income Security Act cannot invest in a manner that hurts expected risk-adjusted returns.

²The word funds here is meant in the broadest sense; What we have in mind is all sort of financial intermediaries that manage clients savings or financial wealth, such as for example mutual funds, sovereign funds, venture capitalists, private equity, banks, etc.

level is particularly inefficient and where the capital search friction is particularly acute. Due to the search friction, concentrating ESG capital in one sector makes it more costly for companies to not comply with the restrictions of the ESGF. The prioritized sector typically does not coincide with the least polluting sector. Above a critical threshold of assets under management, the ESGF diversifies into a second sector. If the ESGF is large enough, first-best can be achieved. Third, we show that the responsible fund can take advantage of the economy's supply-chain network by imposing to the firms it finances restrictions on the choice of their suppliers. This strategy is particularly effective when the sector where reduction in emission would be the most beneficial, say sector i , is also the least subject to the search friction. Firms in this sector can easily substitute ESGF's capital with "non-responsible" capital, which limits the *direct* impact of the ESGF. It is then optimal for the ESGF to invest all its capital in the sector downstream to sector i , say sector j , and impose to firms in that sector to purchase their input from clean producers in sector i . The ESGF has thus an *indirect* impact on sector i who endogenously splits into a mass of clean firms selling to sector j firms, and and dirty firms selling to consumers at relatively low prices. This mechanism is in line with empirical results by Dai et al. (2019) and Schiller (2018) who document propagation of ESG standards along the supply chain network. Finally we extend our model to have investors with heterogeneous strict preference on ESG investing, and costly screening for ESGF to tell "clean" from "dirty" firms. This allows to endogenize the size of the ESGF and leads three Pareto ranked equilibria. In the Pareto superior equilibrium the ESGF asset under management is largest, and if screening cost is small enough, the first best social optimum can be achieved.

Literature Review. Our paper is related to different strands of the literature. On the empirical side, several papers explore the performance and preferences of socially responsible investors. On performance, the evidence is quite mixed. Hong and Kacperczyk (2009) and El Ghouli et al. (2011) document that “sin stocks” have positive abnormal returns suggesting their cost of capital is higher. Bolton and Kacperczyk (2019) also find that stocks of companies with higher CO2 emission intensity earn higher returns. Barber et al. (2018) finds that impact investing private equity earns lower returns; Zerbib (2019) and Baker et al. (2018) find that green bonds are issued at a premium (controlling for risk), hence deliver lower returns. However, there is also evidence in the opposite

direction, arguing that a company’s ESG performance predicts positively its stock-returns. A possible explanation is market under-reaction to ESG information. For example, Edmans (2011) documents that firms that treat their employees well have positive abnormal returns. Derwall et al. (2005) find that more socially responsible portfolios provide higher average returns. Gibson and Krueger (2018) and Henke (2016) find a link between a portfolio sustainability footprint and its performance in the equity and bond markets respectively. Andersson et al. (2016) report over-performance of decarbonized stock indices and predict such green indices will out-perform further in the future: They argue that the market fails to fully recognize the impact of future restrictions on CO2 emissions³. In a broad meta-analysis of the empirical literature on responsible investing, Margolis et al. (2007) concludes that there is an ambiguous correlations between social responsibility and financial returns.

Regarding the motivations of socially responsible investors, Krueger et al. (2018) use a large-scale survey of institutional investors and find that they believe that screening companies based on environmental information can enhance risk-adjusted returns because equity valuations do not fully reflect climate risks. Hartzmark and Sussman (2018) reports a causal link between the flows into mutual funds and the publication of their sustainability ratings. Riedl and Smeets (2017) collect survey data and find that moral preferences are important factors for decisions by this type of investors. In our model, as we want to be conservative, we do not assume that investors are willing to bear lower returns for doing good. In particular, this allows our normative results to be agnostic about the existence or non-existence of a temporary under-reaction of markets to ESG information.

On the theory side, several papers model the implications of the existence of socially responsible investors. For instance, Heinkel et al. (2001) develop a model where a fraction of investors boycott firms that are not clean. “Dirty” companies trade at a discount compared to their “clean” peers, because in equilibrium, their shareholders (i.e. those that have no moral concerns) are more concentrated in “dirty” companies. In our paper, there is no uncertainty which shuts down the channel explored by Heinkel et al. (2001). Morgan and Tumlinson (2019) develop a theory where firms internalize externalities in that they solve a free-rider problem experienced in the production of a

³This view is congruent with that of central bankers such as Matt Carney who have repeatedly warned that climate risks are not fully reflected in asset valuations yet.

public good by maximizing shareholder welfare. Chowdhry et al. (2014) studies optimal contracting in the presence of externalities, when some investors are willing to pay for public goods, providing a foundation for impact securities. In the same spirit, Oehmke and Opp (2019) offer a theory of responsible investing where a moral hazard problem creates financial constraints that interact with externalities. By internalizing social externalities, responsible investors facilitate the scaling of virtuous projects and they are complementary to regular financiers. Different from Oehmke and Opp (2019), in our model, responsible investors have the same returns as regular investors. Our model emphasizes general equilibrium forces and a search friction that endows investors with some bargaining power.

In the following, Section 2 describes our analytical framework. Section 3 compares the laissez-faire equilibrium with the social optimum. Section 4 analyzes the ESGF optimal portfolio and policy when the fund focuses on reducing the emissions solely of the firms it finance. Section 5 analyzes the impact of ESGF can have exploiting the supply chain to curb the emission the firms it finance and/or of their suppliers. Section 7 concludes.

2 Model

We consider a competitive general equilibrium economy where agents are atomistic, enjoy consumption, but suffer from the toxic emissions generated by production of goods. The population of agents is composed of a mass 1 of capitalists and a mass 1 of entrepreneurs. Each capitalist is endowed with one unit of capital but lacks the skill to run a company. Each entrepreneur has the skill to run a company but has no capital. There are 2 goods; each good can be consumed or used as an input to produce the other good. Each good is produced in an industry, $i = 1, 2$, consisting of a continuum of competitive firms (with endogenous mass).

Technology. Let firms of industry i be indexed by $f \in [0, K_i]$, where K_i is the (endogenous) capitalization of industry i . Each firm requires one unit of capital. The quantity $y_{i,f}$ of good i produced by a single firm f from the unit of capital depends on the firm's input quantity $x_{j,f} \geq 0$

of good j and the level $e_{i,f} \in [0, 1]$ of toxic emission the firm releases during production:

$$y_{i,f} = e_{i,f}^{\beta_i} x_{j,f}^{\alpha_{ij}} \quad (1)$$

where $\beta_i \in (0, 1)$ and $\alpha_i \in (0, 1)$. The industry's aggregate emission is $E_i = \int_0^{K_i} e_{i,f} df$.

Preferences. Individuals derive utility from the consumption of both goods, but suffer from the aggregate amount of emissions in the economy. Namely an individual utility from a consumption plan (c_1, c_2) is

$$u(c_1, c_2, E_1, E_2) = \frac{c_1^{\gamma_1} c_2^{\gamma_2}}{(1 + E_1)^{\delta_1} (1 + E_2)^{\delta_2}} \quad (2)$$

where $\gamma_1 + \gamma_2 = 1$, and $\delta_i, i = 1, 2$, measures the disutility due to industry i 's emissions .

Goods markets Goods are exchanged in competitive markets, at prices that we denote $p_i, i = 1, 2$.

ESG policy and compliance conditions. Within this framework we introduce three mutual funds: a fund investing in industry 1, a fund investing in industry 2, and an ESG fund (ESGF henceforth) that can invest in both industries. The ESGF can commit to policies specifying maximal emissions thresholds specific to each industry. Namely, we denote with (\hat{e}_1, \hat{e}_2) the ESG policy. An entrepreneur in industry i *complies* with the ESGF requirements only if her firm's emission $e_{i,f}$ does not exceed \hat{e}_i . In any given industry only the entrepreneurs who comply can be financed by the ESGF. The capital that entrepreneurs raise can come either from the ESGF, which has requirements, or from other investors, which are purely interested in financial performance. We introduce below a search friction in capital markets, which gives to the ESGF an ability to enforce constraining policies on firms. To describe this search friction, it is easier to first explicit the sequence of play in our model.

Sequence of play. The following actions unfold sequentially :⁴

⁴This timing of actions is given for expositional clarity. Because this is a single period general equilibrium economy where production and consumption are simultaneous, strictly speaking the agents interaction is modeled

1. The ESGF announces its policy.
2. Each capitalist choose how to allocate their capital among the three funds.
3. Each entrepreneur chooses irreversibly the good i that she wants to produce and a technology that determines her firm's emissions.
4. Entrepreneurs search for capital.
5. Production happens and output is sold. Profits are split between the entrepreneur and the capitalists: an (exogenous) fraction λ of profits is paid to the entrepreneur and the rest is paid to the capitalists who financed the firm.
6. Individuals spend their revenues to consume.

Search for capital. We now specify the search friction that we introduce in capital markets. Let K_i denote the aggregate amount of capital invested in industry i and S_i be the amount of capital that the ESGF invests into industry i . We define $s_i := \frac{S_i}{K_i}$, the resulting fraction of industry i capital that comes from the ESGF. We assume that there are some frictions in the matching between entrepreneurs and capital that leads to a matching function $\Phi(e_{i,f}, \hat{e}_i)$ indicating the probability of being financed for an entrepreneur in industry i , given the emission level of her firm $e_{i,f}$ and the ESG policy \hat{e}_i in sector i . In the appendix we explicit a standard search game, and show that it leads to the following equilibrium expression for $\phi(\cdot)$:

$$\Phi(e_{i,f}, \hat{e}_i) := \begin{cases} 1 & \text{if } e_{i,f} \leq \hat{e}_i \\ \max \left\{ \frac{1-s_i}{1-\eta_i s_i}, 0 \right\} & \text{if } e_{i,f} > \hat{e}_i \end{cases},$$

where $\eta_i \in [0, 1]$ is an industry specific parameter measuring the fluidity of the capital-entrepreneur matching market. Note that $\Phi(\cdot)$ is 1 for a compliant entrepreneur, reflecting the fact that a compliant entrepreneur can be financed by all types of capitalists. If $\hat{e}_i < e_{i,f}$, then $\Phi(\cdot)$ decreases with s_i reflecting the fact that it becomes more difficult for a non-compliant entrepreneur in industry i to find financing if a larger fraction of the pool of capital dedicated to this industry is ESG. What is important to note is that $\Phi(\cdot)$ spans two intuitive polar cases: For $\eta_i = 1$, $\Phi(\cdot)$ is 1, which means

as a simultaneous move game, where all agents correctly anticipate the other agents's strategies.

that the matching market is frictionless. For $\eta_i = 0$, and $e_{i,f} > \hat{e}_i$, one has $\Phi(\cdot) = 1 - s_i$, which is the fraction of non-ESG capital invested in industry i .⁵ The intensity of the matching friction is measured by $1 - \eta_i \in [0, 1]$. Hence, $\eta_i < \eta_j$ means that capital matching friction is more severe in industry i than in industry j . In this case we say that industry i is the *friction industry*.

Having this timing in mind we can solve the model by backward induction.

Consumption choices. Consider an individual whose revenue is w . Her consumption choice solves:

$$\max_{c_1, c_2} \quad \frac{c_1^{\gamma_1} c_2^{\gamma_2}}{(1 + E_1)^{\delta_1} (1 + E_2)^{\delta_2}} \quad (3)$$

$$s.t. \quad p_1 c_1 + p_2 c_2 \leq w \quad (4)$$

Note that, since they are atomistic, agents take aggregate emissions (E_1, E_2) as exogenously given. Taking the first order condition, the individual's demand for good i is

$$c_i = \frac{\gamma_i w}{p_i}, \quad (5)$$

that brings to her a level of utility

$$u^*(w, E_1, E_2) = w \frac{\left(\frac{\gamma_1}{p_1}\right)^{\gamma_1} \left(\frac{\gamma_2}{p_2}\right)^{\gamma_2}}{(1 + E_1)^{\delta_1} (1 + E_2)^{\delta_2}}. \quad (6)$$

which is linearly increasing in the individual's wealth w .

Production choices. Consider a firm in industry i with a technology inducing emissions $e_{i,f} = e \in [0, 1]$. Then the firm's demand for good j solves

$$\operatorname{argmax}_{x_j} \quad p_i y_i - p_j x_j \quad (7)$$

$$s.t. \quad y_i = e^{\beta_i} x_j^{\alpha_{ij}} \quad (8)$$

⁵Hence $\eta_i = 0$ can be interpreted as a matching technology where the entrepreneur has a unique random draw from the pool of capitalists to find a match.

The resulting demand of good j from this firm is

$$x_j = \frac{\alpha_{ij} p_i y_i}{p_j} \quad (9)$$

and firm's profit is

$$\pi_i(e) = p_i y_i (1 - \alpha_{ij}) = \left(p_i e^{\beta_i} \left(\frac{\alpha_{ij}}{p_j} \right)^{\alpha_{ij}} \right)^{\frac{1}{1-\alpha_{ij}}} (1 - \alpha_{ij}) \quad (10)$$

which is increasing in the level of emission e .

Entrepreneur's choice: sector and technology. An entrepreneur has to choose ex-ante (before raising capital and producing) her firm's sector i and emission level $e \in [0, 1]$.⁶ The entrepreneur spends her revenue to consume. From expression (6), the level of utility she will achieve is linear in her revenue. Thus, an entrepreneur chooses her firm's sector i and the emissions level e such as to maximize her expected revenues. The entrepreneur's revenue equals an (exogenous) fraction λ of the firm's profit $\pi_i(e)$ if she is financed, and zero otherwise.⁷ The probability of finding capital is $\Phi(e, \hat{e}_i)$, which depends on emissions choice e . Hence the maximization program that describes the choice by the entrepreneur of her sector and emission level writes:

$$\max_{i \in \{1, 2\}, e \in [0, 1]} \Phi(e, \hat{e}_i) \lambda \pi_i(e) \quad (11)$$

This maximization trades off between (1) the fact that profits conditional on being financed increase in emissions and (2) the fact that finding financing is less likely if the firm does not comply.

Capitalists' portfolio choice. Consider now a capitalist who has to choose how to allocate his unit of capital among the three funds. As each capitalist is atomistic, he takes the aggregate level of emissions as exogenous and thus chooses his portfolio such as to maximize his revenue. That is, he invest his capital in the funds providing the highest return. Let r_1 , r_2 and r_F denote the respective

⁶The idea here is that when an entrepreneur meets capital providers, she presents all the characteristic of the firm she would like to be financed, i.e. the firm's output and production technology.

⁷Here $\lambda \in (0, 1)$ can be seen as the result of Nash bargaining between the entrepreneurs and the capitalists.

returns on fund 1, fund 2 and the ESGF. Whereas all capitalists' priority is on returns, we assume that an exogenous mass S of capitalists is ESG sensitive, in the sense that they will invest all their capital in the ESGF if and only if $r_F \geq r_1, r_2$. The remaining $1 - S$ capitalists invest in the ESGF if and only if $r_F > r_1, r_2$.

We can now define a competitive equilibrium of this economy

Definition 1 *An equilibrium is a set of prices (p_1, p_2) and fund returns (r_1, r_2, r_F) , such that all agents maximize their utility taking the prices and the ESG policy as given; prices are such that the markets for goods and for capital clear; the ESGF chooses its policy to maximize agents' utility.*

The equilibrium is said to be symmetric if all firms in the same industry choose the same technology.

We normalize prices such that agents' aggregate wealth is 1. The following proposition describes some properties that are common to all symmetric equilibria of this economy.

Proposition 1 *In a symmetric equilibrium of the economy:*

1. *In every industry i either all firms comply or no firm complies.*
2. *The total sales revenue of industry i is equal to*

$$Z_i := \frac{\gamma_i + \alpha_{ji}\gamma_j}{1 - \alpha_{ij}\alpha_{ji}} \quad (12)$$

3. *The capitalization of industry i is $K_i = Z_i(1 - \alpha_{ij})$.*
4. *The return on capital equals $r = 1 - \lambda$, no matter the firm in which the capital is invested.*
5. *All firms realizes the same profits $\pi_i = 1$, $i = 1, 2$. Thus, entrepreneurs are indifferent between producing in industry 1 or 2.*
6. *Individual revenues are $1 - \lambda$ for a capitalist and λ for an entrepreneur.*
7. *Let $e_i := \frac{E_i}{K_i}$ denote the average per-firm emission in industry i . Then the equilibrium level of utility of an individual with revenue w is equal to $U(e_1, e_2)w$, where*

$$U(e_1, e_2) := C \frac{e_1^{\beta_1 Z_1} e_2^{\beta_2 Z_2}}{(1 + K_1 e_1)^{\delta_1} (1 + K_2 e_2)^{\delta_2}} \quad (13)$$

where C is a strictly positive constant.

The proposition shows that the equilibrium has three remarkable properties. First, the equilibrium composition of the market portfolio, and hence the size K_i of each industry $i = 1, 2$, only depends on consumers' taste for the two goods (γ_1 and γ_2) and the goods productivity as intermediary goods (α_{12} and $\alpha_{2,1}$). Second, the equilibrium level of utility equals $U(e_1, e_2)\lambda$ for an entrepreneur and $U(e_1, e_2)(1 - \lambda)$ for a capitalist hence we can identify social welfare with $U(e_1, e_2)$. Third, all funds provide exactly the same return no matter whether they are ESG or not. Hence in equilibrium the amount of capital invested through the ESGF is S , that is the total capital owned by ESG sensitive capitalists.

3 Levels of Emission: Laissez-Faire vs. First Best

3.1 Laissez-Faire

We call laissez-faire the equilibrium that prevails absent the ESG fund. Because firms are price-takers, each firm's profit is increasing in the amount of its emission. Hence, absent any incentive or regulation, all firms set emissions at maximum level, that is $e_1 = e_2 = 1$ for all firms. The social welfare is then $U(1, 1)$.

3.2 First-Best

If consumers suffer strongly enough from an industry's aggregate emission, it is socially optimal to put a cap on firm's emission in that industry. Note that because emissions are necessary for production, a 0-emission level cannot be socially optimal. Consider a benevolent planner who can choose the level of emission in each firm as to maximize agents' utility. Formally the first best social optimum solves

$$\max_{e_1, e_2} U(e_1, e_2)$$

Then we have

Proposition 2 *The social optimum is attained iff each firm in industry i emits*

$$e_i^* = \min\left\{\frac{\beta_i}{(\delta_i - \beta_i Z_i)(1 - \alpha_{ij})}, 1\right\} \quad (14)$$

The socially optimal level of emission results from the tradeoff between the discomfort of emission on consumer's utility, measured by δ_i , and the productive advantage of emission for good i . The latter increases with β_i , the production elasticity of emission, with α_{ij} and α_{ji} , the production elasticity in the input output matrix, and with γ_i , the utility elasticity from consuming good i . Thus the laissez faire equilibrium is sub-optimal if for some i , $\delta_i > \frac{\beta_i(1+K_i)}{1-\alpha_{ij}}$.

4 Impact and Optimal ESG strategy

In this section we characterize the optimal strategy that the ESGF should implement to maximize agents utility.

Impact of the ESGF. We define the impact of a policy of the ESGF as the difference in social welfare when the fund applies this policy vs. when the fund does not exist (or equivalently when the fund does not impose restrictions). We first show that his consequentialist definition of impact implies that tilting the sector allocation of the fund has by itself no impact on the economy. To have impact, the ESGF needs to impose limits to the emissions of firms where it invests.

4.1 Can industry tilting have impact by itself?

In our model, the answer is no. The mere shifting of a portfolio toward less polluting industries has no impact:

Corollary 1 *Suppose the ESGF imposes no emission restriction to the firms it finances, i.e., $\hat{e}_1 = \hat{e}_2 = 1$. Then, no matter the portfolio composition of the ESGF, $e_1 = e_2 = 1$ and individual utility is $U(1, 1)$.*

There are two reasons for this result: First, portfolio tilting cannot change the composition of the market portfolio. From point 2 of Proposition 1, the composition the relative size of each

industry, only depends on the consumer taste γ_1, γ_2 and the input-output matrix $(\alpha_{12}, \alpha_{21})$. Thus, in the equilibrium the flow of non-ESG capital would undo the tilt in the ESG one.⁸ Second, if to be financed by the ESGF, entrepreneurs do not need to reduce emissions, they just choose to maximize profit by setting them to maximum level, as in the laissez-faire case.

Corollary 1 implies that to have an impact the ESGF has to impose a restrictive emission policy on the firms it finances. The emission caps that the ESGF can impose, as well as the optimal composition of its portfolio, both vary with the size of its portfolio. This is what we want to characterize next.

4.2 Can the ESGF impose limits to emissions?

We now characterize how far the ESGF can go in imposing limits to emissions, as a function of the capital amount it invests in a given industry. The intuition is that the minimum emission ESGF can impose is the \hat{e}_i such that entrepreneurs are indifferent between complying or not. This happens if expected profits without complying (and thus setting emissions to 1) equal expected profits conditional on compliance:

$$\Phi(1, \hat{e}_i)\pi_i(1) = \pi_i(\hat{e}_i), \quad (15)$$

Equation (16) highlights the key role played by the matching friction in the ability of the ESGF to impose emission limits: if capital markets are perfectly fluid, $\Phi(1, \hat{e}_i) = 1$, then $e_i = 1$ is the only solution of (16). The economic intuition is that matching frictions make entrepreneurs worry about being compatible with the ESGF, in case they are matched with it. Also, note that the matching friction enables the ESGF to affect the behavior of *all* firms in industry i , even though it finances only a fraction of them. This is because emission choices are made ex-ante. In turn, this guarantees that the returns from the ESGF are competitive with those of other funds: In a sense, non-ESG investors are involuntarily ESG-compliant in our model, as the ESGF affects the behavior of all entrepreneurs in the same industry.

⁸A situation in which $S > K_i$ and the ESGF invests all this capital in industry i cannot occur in equilibrium because it would lead to profits of firms in industry i being lower than for the other industry j . Hence either the return on the ESGF will be strictly smaller than the return on fund j or entrepreneurs' revenue in industry i will be strictly lower than in industry j , which cannot be an equilibrium.

By developing equation (16), we can explicit the minimum amount of capital the ESGF has to invest in an industry as a function of the desired emission limits in that industry:

Lemma 1 *The minimum amount of capital that the ESGF needs to invest in industry i to successfully impose a limit to emissions \hat{e}_i is:*

$$S_i(\hat{e}_i) = \frac{1 - \hat{e}_i^{\frac{\beta_i}{1-\alpha_{ij}}}}{1 - \eta_i \hat{e}_i^{\frac{\beta_i}{1-\alpha_{ij}}}} K_i \quad (16)$$

By pledging $S_i(\hat{e}_i)$ to industry i and committing to finance firms in that industry only if their emission does not exceed \hat{e}_i , the ESGF induces all firms in industry i to reduce their emissions to \hat{e}_i .

$S_i(\hat{e}_i)$ is larger when \hat{e}_i is smaller. This means that when the ESGF increases the capital it invests in an industry, it can impose tighter emission requirements in that industry: This is because entrepreneurs know they are more likely to be matched with the ESG investor and hence are more inclined to comply. For the same reason, the ability to reduce an industry's emission is stronger in industries that are small (i.e, K_i is small) and where the capital matching friction is high (i.e. η_i is small). It is also easier to reduce emissions when β_i is low, as the entrepreneur sacrifices less output by complying. We can express the constrained maximization problem of the ESG fund managing an amount of capital S as follows:

$$\begin{aligned} \max_{e_1, e_2} \quad & U(e_1, e_2) \\ \text{s.t.} \quad & S_1(e_1) + S_2(e_2) \leq S \end{aligned}$$

This makes apparent that there is a tradeoff between limiting emissions in one industry versus the other. The tradeoff comes from the fact that to impose lower emissions to industry i , the ESGF needs to increase the capital it allocates to that industry at the expenses of industry j , reducing in this way its grip on industry j 's emissions.

What should the ESGF do when it manages a small fraction of the total capital? One can

see that, instead of spreading capital thin on the two sectors, it should instead concentrate capital in one sector. The sector to be prioritized is the one where the marginal impact of capital is the strongest.

Lemma 2 *priority to the highest impact sector.* *If S is small, the ESGF invests all its capital in only one sector. This sector is the one where capital has the highest marginal impact on welfare: $i_0 = \operatorname{argmax}_{i \in \{1,2\}} \left(\frac{1-e_i^*}{e_i^*} \right) \left(\frac{1-\eta_i}{1+K_i} \right)$*

The expression for i_0 follows from simple computations, and has a clear economic interpretation: To determine i_0 , the industry on which a small ESGF should focus, two elements need to be considered, the social desirability in reducing emission, measured by $(1 - e_i^*)/e_i^*$, and the effectiveness of ESG incentives on entrepreneurial choice, measured by $(1 - \eta_i)/(1 + K_i)$. Given the same first best emission level, i.e., $e_1^* = e_2^*$, the ESGF should first focus on where its investment is most influential. Given the same effectiveness, the ESGF should first focus on the industry in which reduction of emission is most desirable, that is the critical industry, i.e. where e_i^* is the smallest. This suggests that rather than focussing on liquid shares of companies, impact investing should prioritize primary offerings, private equity, as well as less liquid stocks.

We can now characterize fully the ESGF's portfolio composition and policy that must be chosen in order to maximize social welfare:

Proposition 3 *Let S be the size of the fund and $S^* := S_1(e_1^*) + S_2(e_2^*)$. Consider ESG policies that aim to maximize social welfare by only constraining firms direct emissions. There is $\underline{S} \in (0, S^*)$ such that all firms comply with the ESG policy, and:*

1. *If $S \leq \underline{S}$, then the ESGF invests only in industry i_0 and imposes emissions in that industry to be lower than $\left(\frac{K_{i_0}-S}{K_{i_0}-\eta_{i_0}S} \right)^{\frac{1-\alpha_{i_0}}{\beta_{i_0}}}$.*
2. *If $\underline{S} < S < S^*$, then the ESGF invests in both industries.*
3. *If $S \geq S^*$, then the ESGF invests in each industry i at least $S_i(e_i^*)$ its policy imposes first-best emissions: $(\hat{e}_1, \hat{e}_2) = (e_1^*, e_2^*)$.*

Let us interpret the different elements of Proposition 3:

When the fund size is particularly small, that is $S < \underline{S}$, the ESGF concentrates capital in industry i_0 , as was discussed above in Lemma 2. Firms in industry i_0 will comply, whereas in the other industry all firms will set emission at the maximum, $e_j = 1$. As more and more capital is invested in i_0 , the marginal impact of capital in that industry goes down (since \hat{e}_i gets closer to e_i^*), getting closer to that in the other sector. The threshold \underline{S} corresponds to the mass of ESG capital that needs to be invested in i_0 , such that the marginal impact of incremental ESG capital is the same in each sector.

For $\underline{S} < S < S^*$, the ESGF invests in both sectors. It equalizes the marginal impact of capital in each of the two sectors. The size of ESGF is however not sufficient to bring emissions to the first best.

When the size of the ESGF $S > S^*$, the fraction of the total capital managed by the ESGF is large enough for the fund to be able to induce all firms to comply with the first best. That is, the ESGF invests in both industries an amount sufficient to make the policy (e_1^*, e_2^*) acceptable to all entrepreneurs. Note that when $S > S^*$, the marginal impact of additional ESG capital is zero, as the first-best is already implemented. An increase in the level of the capital market friction, reduces the total amount of ESG capital that is necessary to reach the first best.

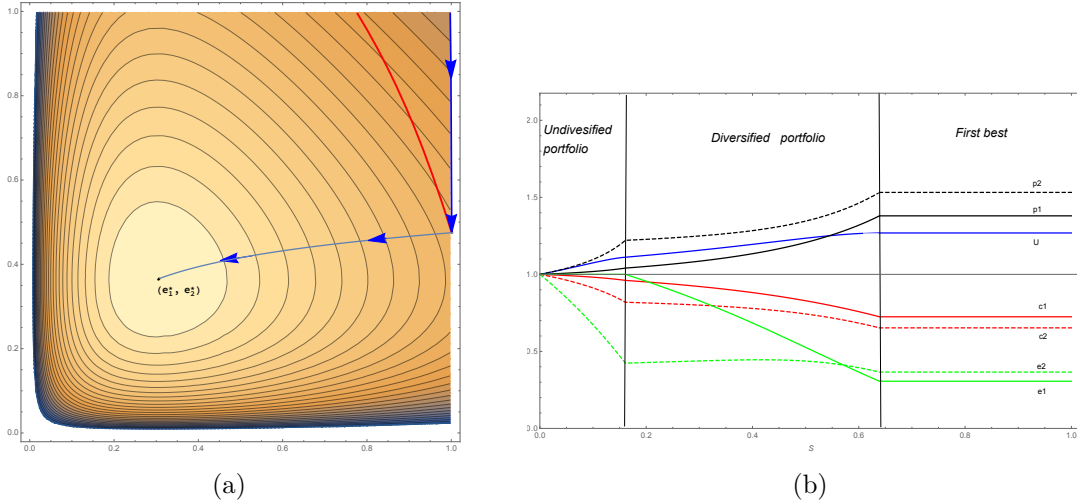


Figure 1: Panel A shows ESG maximization problem in the plane (e_1, e_2) . The black curves are iso-social-welfare curves. The red curve indicates the minimum levels of (e_1, e_2) that can be achieved when $S = \underline{S}$. The blue line indicates the constraint socially optimum level of emission for the different $S \in [0, 1]$ where arrows move from $S = 0$ (top-right corner) toward $S \geq S^*$, (point (e_1^*, e_2^*)). Panel B presents the ratio between macroeconomic variables and their level in the laissez-faire situation as a function of the size S of ESGF: Social welfare (blue line), consumptions (red lines), emissions (green lines), goods prices (black lines). The kinks occur at $S = \underline{S}$ and at $S = S^*$.

The next corollary relates the size of the ESGF with the level of utility, the level of consumption and the level of price.

Corollary 2 *As long as $S < S^*$, an increase in S brings an increase in the individuals' utility level and in the goods prices. It decreases the production and consumption of each good and weakly decreases level of emission in each industry. For $S \geq S^*$, utility and prices are maximal, whereas production and consumption are minimal.*

This result sheds light on the fact that in our model social welfare and aggregate consumption are decoupled: the ESGF helps reaching a higher level of welfare by implementing a lower level of aggregate consumption. The reason is that reducing emissions leads to a loss of productive efficiency, hence to lower aggregate output.

4.3 Footprint vs Impact of the ESG fund

A notion that is often used in practice is the ESG footprint of a portfolio, which measures if a portfolio is tilted towards companies that have important levels of externalities. For instance, the “carbon footprint” of a portfolio measures the average level of emissions per unit of capital of companies in the portfolio. In real-world implementation of ESG investing, a relatively usual approach consists in limiting the “Carbon footprint” of the investment portfolio.⁹ However, in our set-up, this approach is potentially highly misleading. In fact, it turns out that there are cases where the toxic footprint of an impact maximizing ESG fund would be *higher* rather than lower than that of regular funds. The reason is that to maximize their impact, ESGF should focus their investments in industries where they can convince managers to implement changes that are highly beneficial for welfare. In particular, investing in an industry that does not pollute is simply useless in terms of impact and consumes some of the ESGF impact capacity in other sectors.

Formally, we can define the toxic footprint of a portfolio allocated with industry weights (ω_1, ω_2) as: $\delta_1 \omega_1 e_1 + \delta_2 \omega_2 e_2$. The definition of the toxic footprint can be understood by going back to the

⁹This type of approach is also sometimes recommended by academics: For instance Gollier (2019) proposes that ESG funds should report their performance by subtracting from financial returns a multiple of the carbon emissions, where the multiple would be an explicit carbon price.

utility function defined earlier.¹⁰ Then we have:

Proposition 4 *The ESG fund does not necessarily have a better footprint than that of a regular fund.*

The proof consists in finding a simple example: For instance, if $\delta_1 = 0$ and $\delta_2 > \frac{\beta_2(1+K_2)}{1-\alpha_{21}}$, the ESG fund will be all invested in sector 2 (sector 1's emissions are not harmful but sector 2's are); whereas the “regular” investor is diversified across both sectors. The proposition highlights that it is important to distinguish between footprint and impact, a distinction that is not always clear in the debate.

5 Using both direct and indirect emissions caps

In this section we explore what happens when the ESGF can express restrictions not only on the emissions of the firms where it invests, but also on their suppliers.

5.1 Internally consistent policies

Consider a firm in industry i . Beside its emission e_i directly resulting from production, the firm's economic activity is associated with the direct emission of the firm's supplier of good $j \neq i$. We call this the firm's *indirect emission* and denote it with e_{Ui} . In this section we study the impact the ESGF can have when eligibility to ESG capital encompasses direct and indirect emissions. That is, an entrepreneur in industry i complies with the ESGF requirements only if both her firm's direct and indirect emissions do not exceed the caps set by the ESGF. We focus on ESGF policies that are internally consistent, meaning that a firm in industry i which complies, is able to sell its output and purchase its input to and from compliant firms in industry j , respectively. Formally,

Definition 2 *A consistent policy is a quadruple $\hat{e} = \{\hat{e}_1, \hat{e}_{U1}, \hat{e}_2, \hat{e}_{U2}\} \in [0, 1]^4$, such that $\hat{e}_i \leq \hat{e}_{jU}$, for all $i = 1, 2$ and all $j \neq i$*

¹⁰Fix the consumption level; the log utility is up to a constant $\delta_1 \ln(1 + K_1 e_1) + \delta_2 \ln(1 + K_1 e_1)$; The marginal impact on this quantity of a portfolio dk allocated with weights (ω_1, ω_2) is up to a scaling factor $(\sum \delta_i \omega_i e_i) dK$.

Because compliant firms in industry i can only buy from industry j producers whose direct emissions do not exceed \hat{e}_{U_i} , a consistent policy implies the presence of goods markets that are specific to the producers' direct emissions levels. To take this into account we amend our base model in three dimensions. First, prices of goods vary depending on whether their production is compliant with the ESGF policy or not: We assume that if, within an industry i , a strictly positive mass of firms choose the same level e of emission, these firms will sell their outputs in a dedicated competitive market at a price that we denote $p_i(e)$. Note that a firm whose direct emission differs from all other firms has no dedicated market for its output. Because firms are atomistic, we assume that such a firm will be able to smuggle its production and buy its input in any of the markets for the corresponding goods. Second, when the same good is available in more than one market, consumers have the choice of where to purchase the good. Because a single individual's choice has no impact on aggregate emissions, each agent will purchase her consumption goods in the markets where they are the cheapest. Third, because eligibility to ESG capital concerns both direct and indirect emission, when searching for capital the entrepreneur has to specify both its firm direct and indirect emissions.

The following proposition shows that the use of indirect emission caps gives another reason for the ESGF to concentrate capital in only one of the two industries:

Proposition 5 *By adopting a consistent policy and investing in both industries, the ESGF cannot have more impact than when adopting the optimal direct emission policy described in Proposition 3.*

The logic behind this result is as follows. When the ESGF invests in both industries, all firms in each industry comply to the internally consistent policy \hat{e} . If all firms in the other industry j comply, then the only thing firms in industry i need to do in order to comply is to reduce their direct emissions. Hence we are back to the case where the ESGF does solely focus on direct emissions.

5.1.1 Indirect incentives

Can the ESGF have a stronger impact by investing all its capital in a single industry, but requiring the firms it finances to reduce both direct and indirect emissions?

Such a strategy provides to each industry incentives of a different nature. By focusing its capital on a single industry i , the ESGF maximizes its grip on that industry that results from the capital matching frictions. The presence of compliant firms in industry i gives rise to an endogenous mass of firms in industry j who choose to reduce their direct emissions. They do not do this to have better chances to be financed (there is no ESG capital in industry j), but rather because good j produced with a low emission technology trades for a higher price than the same good produced with high emission. As we show in the next Proposition, in equilibrium the industry that receives ESGF capital will only be composed of compliant firms, whereas in the other industry both low-emission and high-emission firms co-exist.

To approach the equilibrium, a first step is to understand the trade-off perceived by a firm in industry i . Using Equation (10), we find that a firm in industry i expects higher profits from compliance than non-compliance if :

$$\underbrace{\left(\frac{\hat{e}_i^{\beta_i}}{(p_j(\hat{e}_{Ui}))^{\alpha_{ij}}} \right)^{\frac{1}{1-\alpha_{ij}}}}_{\text{profitability if comply}} \geq \underbrace{\Phi(1, \hat{e}_i)}_{\text{probability to receive capital if doesn't comply}} \underbrace{\left(\frac{1}{(p_j(1))^{\alpha_{ij}}} \right)^{\frac{1}{1-\alpha_{ij}}}}_{\text{profitability if doesn't comply}} \quad (17)$$

Complying with the ESGF policy involves two costs: the cost of not being able to use maximum emissions when producing, and the cost of using compliant inputs, that are more expensive than non-compliant inputs. This second cost comes from the fact that in equilibrium $p_j(1) < p_j(\hat{e}_{Ui})$. This inequality is itself implied by the coexistence in industry j , of firms producing compliant goods and non-compliant goods: These firms must be making identical expected profits. Given that they face identical prices for input i , this can be expressed, going back to Equation (10) as:

$$p_j(1) = (\hat{e}_{Ui})^{\beta_j} p_j(\hat{e}_{Ui}) \quad (18)$$

By combining Equations (17) and (18), and using the expression of $\Phi(1, \hat{e}_i)$, we get the condition under which firms in industry i prefer to comply.

Condition 1

$$\hat{e}_i^{\beta_i} \hat{e}_{U_i}^{\beta_j \alpha_{ij}} \geq \left(\max \left\{ 0, \frac{K_i - S}{K_i - \eta_i S} \right\} \right)^{1 - \alpha_{ij}}$$

This condition determines the feasible policies $(\hat{e}_i, \hat{e}_{U_i})$ as a function of S and allows us to characterize the equilibrium:

Proposition 6 *Suppose the ESGF only invests in industry i , requiring compliant firms to reduce their direct and indirect emissions to respectively \hat{e}_i and \hat{e}_{U_i} , fulfilling Condition 1. Then, in equilibrium:*

1. *In industry i all firms comply by setting their direct emission at $e_i = \hat{e}_i$ and buying from industry j firms with direct emission of $e_j = \hat{e}_{U_i}$.*
2. *Industry j splits into a mass of size $K_j \theta_j$ of high-emission firms, and a mass of size $K_j(1 - \theta_j)$ of low-emission firms, where $\theta_j := \frac{\gamma_j}{Z_j} \in (0, 1)$. A high-emission (resp. low-emission) firm's direct emission equals 1 (resp. \hat{e}_{U_i}).*
3. *Equilibrium prices for good j satisfy $p_j(1) = (\hat{e}_{U_i})^{\beta_j} p_j(\hat{e}_{U_i}) \leq p_j(\hat{e}_{U_i})$.*
4. *Consumers buy good j exclusively from high emission firms, whereas industry i firms buy input j exclusively from low emission firms.*
5. *Average emission levels per firm are $e_i = \hat{e}_i$ in industry i and $e_j = \theta_j + (1 - \theta_j)\hat{e}_{U_i}$ in industry j .*
6. *Social welfare is proportional to*

$$U_I(e_i, e_j) := C \frac{e_i^{\beta_i Z_i}}{(1 + e_i K_i)^{\delta_1}} \frac{\left(\frac{e_j - \theta_j}{1 - \theta_j} \right)^{\beta_j \alpha_{ij} Z_i}}{(1 + e_j K_j)^{\delta_2}} \quad (19)$$

By providing capital only to industry i and requiring compliant firms in this industry to reduce their direct and indirect emissions to \hat{e}_i and \hat{e}_{U_i} , respectively, the ESGF brings the direct emission to each individual firm in industry i to \hat{e}_i . Note however that the indirect emission cap on industry i only affects the direct emissions of a fraction $1 - \theta_j$ of industry j . The remaining $K_j \theta_j$ firms will set their emission to 1. Thus, the average per-firm emission for industry j is equal to $\theta_j + (1 - \theta_j)\hat{e}_{U_i}$. We can use this expression to translate Condition 1 into the constraint on the average per-firm emission that the policy can induce. This leads to the following maximization problem for the ESGF:

$$\begin{aligned} \max_{e_1, e_2, i \in \{1, 2\}} \quad & U_I(e_i, e_j) \\ \text{s.t.} \end{aligned} \quad (20)$$

$$e_j \geq \theta_j \quad (21)$$

$$e_i^{\beta_i} \left(\frac{e_j - \theta_j}{1 - \theta_j} \right)^{\beta_j \alpha_{ij}} \geq \left(\max \left\{ 0, \frac{K_i - S}{K_i - \eta_i S} \right\} \right)^{1 - \alpha_{ij}} \quad (22)$$

It is worth interpreting constraints (21) and (22). No matter the ESG policy (\hat{e}_i, \hat{e}_i^U) , a fraction θ_j of firms in industry j are setting emissions to 1. Thus, the minimum average emission in industry j cannot fall below θ_j , hence constraint (21). Note that if (22) holds with equality, then e_i must be decreasing in e_j . That is, the stricter the restrictions on industry j , the softer the restrictions applying to industry i need to be. The tradeoff between the emission caps in the two industries is of different nature than that resulting from the purely direct emission policy we explored in Section 4. Here, in order to decrease average emission of industry j , e_j , the ESGF has to lower \hat{e}_{Ui} . This decreases the direct emission for a low-emission firm of industry j . To choose to lower their emissions, these firms must be compensated with a bigger selling price for their product, $p_j(\hat{e}_{Ui})$, compared to the price $p_j(1)$, at which high-emission firms in the same industry can sell theirs. That is, the lower e_j , the larger the relative price $p_j(\hat{e}_{Ui})/p_j(1)$. Thus decreasing e_j increase the cost of input for complying firms in industry i . As a result industry i entrepreneurs choose to comply only if the cap on their direct emission \hat{e}_i is not too small. Thus the negative relation between e_i and e_j implied by constraint (22). As for the direct emission policy, the r.h.s. of (22) shows how the grip of the ESGF on emissions increases with S and decreases with η_i .

5.1.2 Direct incentives vs indirect incentives for a small size ESGF

Recall that we defined the *friction industry* as the industry where η_i is the smallest, and the *critical industry* the one where e_i^* is the smallest. We have seen in Lemma 2 that for S small enough the ESGF's optimal direct emission policy consists in investing all its capital in a single industry. Depending on the social desirability in reducing emission, and the effectiveness of ESG incentives

resulting from capital matching frictions, the industry to prioritize may be the friction and/or the critical industry.

We show below that a small ESGF recurring to indirect incentives should separate the investment choice from its emission cap policy.

Lemma 3 *Assume the ESGF is managing an amount of capital S close to 0. To maximize its impact the ESGF should invest all its capital in the friction industry and adopt a policy focussed solely on reducing the critical industry's emission.*

When the friction industry and the critical industry are the same industry i , this is achieved by imposing only a direct emission cap on the the friction industry.

$$\hat{e}_i = \left(\frac{K_i - S}{K_i - \eta_i S} \right)^{\frac{\beta_i}{1-\alpha_{ij}}}.$$

When the friction industry is i and the critical industry is $j \neq i$, this is achieved by imposing only an indirect emission cap to the friction industry.

$$\hat{e}_{U_i} = \left(\frac{K_i - S}{K_i - \eta_i S} \right)^{\frac{1-\alpha_{ij}}{\beta_j \alpha_{ij}}}.$$

5.1.3 Direct incentives vs indirect incentives for a medium size ESGF

In this sub-section, we want to study more generally if focusing on a single industry's direct and indirect emissions can increase welfare more than focusing solely on direct emissions (of both industries). Clearly if $S \geq S^*$, then the first best can be achieved by investing in both industries. Thus focusing on a single industry can maximize the ESGF's impact only if its size S is relatively small. In the following proposition we provide sufficient conditions under which investing in a single industry and constraining both its direct and indirect emissions is optimal.

Proposition 7 *Suppose $S < S^*$. If $\eta_j - \eta_i$ and/or $\alpha_{ij} - \gamma_j$ large enough, then to maximize its impact the ESGF has to invest S into industry i only and impose to this industry both direct and indirect emission caps.*

The case $\eta_j > \eta_i$ with η_j large corresponds to a situation in which there is some friction in the matching capital market of industry i whereas in industry j capital market is virtually frictionless. In this case the ESGF capital in industry i provides substantially stronger financial incentives than in industry j . To maximize impact, the ESGF should invest all its capital where it can affect firms' production choices and require these firms to purchase from clean suppliers.

The case $\alpha_{ij} - \gamma_j$ large correspond to a situation in which $\alpha_{ij} \simeq 1$ and $\gamma_j \simeq 0$. That is, consumers derive utility mostly from good i rather than from j , but good j represents a substantial input for good i production. In this case the ESGF should invest all its capital in industry i , the 'consumption good' industry, and require this industry to purchase the 'intermediary good' j from clean producers. More specifically, observe that K_i is a decreasing function of a_{ij} . When K_i is small, by investing all its capital in industry i then ESGF acquires a substantial control of the industry and can impose strong limit to direct and indirect emission. The limit on industry i indirect emission will affect only a fraction $1 - \theta_j$ of firms in industry j . However for γ_j close to 0, industry j good is mostly used as an input for industry i , implying that θ_j is close to 0.

6 Endogenizing the size of ESGF

We have so far taken the size S of the ESGF as exogenous. Our motivation for this modeling choice is that institutional factors (such as the existence of a sovereign wealth fund or the legal constraints of pension funds) are likely determinants of the size of the ESGF in a given country. Therefore, we wanted our baseline model to be centered on the issue of *what impact is achievable* for a given S . We now present a natural extension of our model where we endogenize S . To do so, we amend our model along two dimensions. First, we drop the assumption of lexicographic preferences for capitalists such as to allow for a real individual trade-off between financial performance and impact. Second, we assume that verifying the compliance of entrepreneurs with its own standards is costly for the ESGF.

Preferences for impact: Specifically, let us assume that investing in the ESGF adds to a capitalist's utility in proportion to the actual impact I of the ESGF in the economy. Hence, a capitalist's

utility is a function of her consumption plan (c_1, c_2) , the aggregate emissions (E_1, E_2) , the ESGF's impact I , and the fraction ω_{ESG} of her capital invested in the ESGF:

$$\frac{c_1^{\gamma_1} c_2^{\gamma_2}}{(1 - E_1)^{\delta_1} (1 - E_2)^{\delta_2}} + \mu \omega_{ESG} I \quad (23)$$

where, as before, impact is defined as the difference between the levels of social welfare with the ESGF and in laissez-faire. The parameter $\mu \in [0, 1]$ measures the capitalist's sensitivity to ESGF impact. For $\mu = 0$, we are back to our base model. To keep the algebra simple, we assume that μ is uniformly distributed on $[0, 1]$ in the population of capitalists.

Verification costs: We assume that for telling apart compliant from non-compliant entrepreneurs, the ESGF has to hire clerks.¹¹ Namely, the additional management cost incurred by one unit of capital to a compliant firm is $(1 - \lambda)\phi$, where $\phi > 0$ is an exogenous parameter. We already know that in equilibrium a firm's profit equals 1. For a firm that is financed with ESGF capital, this profit will be split as follows: λ goes to the entrepreneur, $(1 - \lambda)\phi$ to the ESGF employees who screened the firm, and $(1 - \lambda)(1 - \phi)$ to the investors who provided the capital. Standard funds do not have screening costs, hence firms financed via these funds generate a return of $(1 - \lambda)$. Thus a capitalist's revenue from investing ω_{ESGF} in the ESGF and the $1 - \omega_{ESGF}$ in standard funds is:

$$\omega_{ESGF} \underbrace{(1 - \lambda)(1 - \phi)}_{\text{ESGF return}} + (1 - \omega_{ESGF}) \underbrace{(1 - \lambda)}_{\text{standard fund return}} = (1 - \lambda)(1 - \phi \omega_{ESGF}).$$

We can consider now the equilibrium size of the ESGF as resulting from two curves: the impact constraint and the supply of ESG capital.

Impact constraint : After the individual choice of how much to invest in the ESGF is made, the ESGF policy, for a given S , is to maximize impact as described in the previous sections.¹² Let $\hat{U}(S)$ denote the social welfare achieved by the ESGF of size S . We can define the *impact constraint* as

¹¹We assume here is that ESGF employees have the same utility function as entrepreneurs. Thus, transfer of wealth from capitalist who inves in ESGF to ESGF's clerks will not change the aggregate demand for goods.

¹²Interestingly, the results are identical if the ESGF goal is to maximize its size S .

follows:

$$I = \hat{U}(S) - U(1, 1) \quad (24)$$

Note that from the previous analysis we know that the r.h.s. is continuously increasing in S , it equals 0 for $S = 0$ and it reaches a plateau as first-best is implemented for $S \geq S^*$.

Supply of S : Consider a type μ capitalist who invests a fraction ω_{ESG} of her capital in the ESGF. She takes S and I as given. Recall that if the social welfare reaches some level U , then in equilibrium, an individual with revenue w achieves a utility from consumption equal to Uw . When choosing ω_{ESG} , a capitalist maximizes the following linear function

$$\underbrace{\hat{U}(S)}_{\text{capitalist's utility from consumption}} \underbrace{(1-\lambda)(1-\phi\omega_{ESG})}_{\text{capitalist's revenue}} + \underbrace{\mu\omega_{ESG}I}_{\text{capitalist's utility from ESG investing}} \quad (25)$$

Differentiating with respect to ω_{ESG} , we find that capitalists who have a type $\mu \geq \frac{\phi(1-\lambda)\hat{U}(s)}{I}$, invest all their capital in the ESGF (i.e., $\omega_{ESG} = 1$), whereas the others set $\omega_{ESG} = 0$. Now, given that μ is uniformly distributed in the population of capitalists, the supply of capital gathered by the ESGF is:

$$S = \max\left\{1 - \phi \frac{(1-\lambda)\hat{U}(S)}{I}, 0\right\} \quad (26)$$

The economic intuition is that the supply of ESG capital increases with impact I and that there is a critical level of impact below which $S=0$ as investors find impact generated by ESGF too small compared to the performance cost implied by ϕ .

Equilibrium level of S : The equilibrium level of S must be such that the supply function and the impact constraint match, as depicted in Figure 2. That means, combining Equations (24) and (26), that is, S is solution of:

$$S = \max\left\{1 - \phi \frac{(1-\lambda)\hat{U}(S)}{\hat{U}(S) - U(1, 1)}, 0\right\} \quad (27)$$

Let $\phi^* := \frac{(1-S^*)(\hat{U}(S^*)-U(1,1))}{(1-\lambda)\hat{U}(S^*)} > 0$. Then we have

Proposition 8 1. *If ϕ is large enough then $S = 0$.*

2. If ϕ is small enough then generically there are at least two distinct equilibrium levels of S , beside $S = 0$.
3. If $\phi \leq \phi^*$, then the first best social optimum can be achieved.

The economic forces leading to multiple equilibria is a form of strategic complementarity between ESG investors: If more investors invest in the ESGF, the ESGF has more impact, which in turn makes investing in the ESGF more attractive. Note that the different equilibria are Pareto ranked both the smallest and the largest are stable equilibria.

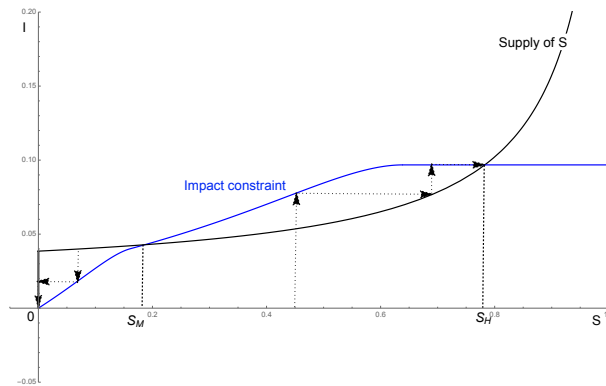


Figure 2: This figure shows the two curves defined in equations (26) and (24) in the plane S, I . The *Supply* curve in black reflects the fact that more investors are willing to invest in the ESGF when impact is large. The *Impact* curve in blue reflects the fact that the impact that the ESGF can achieve increases with the total assets that it manages. S_H and S_L are the Pareto superior and Pareto inferior equilibrium levels of S . The arrows indicate the evolution of I and S off equilibrium.

7 Conclusion

This paper develops a general equilibrium model of a productive economy with negative externalities. We analyze the strategy of a ESG fund which aims at maximizing social welfare, when all individuals and firms do not internalize the externalities of their choices. We show that if capital markets are subject to a search friction, this ESG fund can raise assets and improve social welfare despite the fact that all agents are selfish. The ESG fund has an impact on companies' behavior, forcing them to partially internalize externalities. We derive the fund's optimal policy in terms of industry allocation and pollution limits imposed to its portfolio companies. The ESGF finances firms in exchange of some reduction in their direct and/or indirect emission.

We show that the fund applies a pecking order: It prioritizes investments in companies where (i) the inefficiency induced by the externality is particularly acute and (ii) the capital search friction is strong. We also show how the ESG fund can take advantage of the supply-chain network: It can amplify its impact by focusing on a single industry but imposing restrictions on the suppliers used by the firms where it invests. Our model can be used to make several policy and practical recommendations: The first one is that ESG investing should coordinate on common standards and prioritize a few sectors of intervention. It should focus on segments where markets are less liquid (private equity, primary offerings, small caps). A second important practical message is that impact cannot be assessed via carbon footprint. An ESGF that has a passive strategy consisting in investing in the less polluting industries has no impact on the level of emission nor on the social welfare. Our model also points at the importance of regulations allowing reliable firm-level information on direct and indirect emissions. Last, at times where public capital is needed by private actors, the government should act as ESGF (i.e. set optimal constraints on firms).

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Appendix 1: Proofs

Proof of Proposition 1.

1. Let first prove that in equilibrium in any given industry either all firms comply or no firm complies. Suppose that there is an equilibrium where in industry i compliant and non-compliant firms coexist. Note first that each compliant firm in industry i will set its emission at $e_{i,f} = \hat{e}_i$. In facts setting emission below \hat{e} will not increase the entrepreneur probability of being financed whereas it will decrease the firm's total output and profit. For the same reason a non-compliant firm in industry i will set its emission to $e_{i,f} = 1$, rather than to any other level below 1. Let denote with $\pi_{i,c}$ and $\pi_{i,n}$ the profits of a compliant and a non-compliant firm respectively. Note that from (10), one has $\pi_{i,c}/\pi_{i,n} = \hat{e}_i^{\beta_i} \leq 1$. If $\hat{e}_i = 1$, then there is no difference between a compliant and a non-compliant firm. If $\hat{e}_i < 1$, then $\pi_{i,c} < \pi_{i,n}$. Because one unit of capital invested in a firm with profit π provides $\pi(1 - \lambda)$ to the investor, the return on capital on non-compliant firms will be strictly larger than the return on capital on compliant firms. This implies that the average return on industry i will be strictly larger than the return on compliant firms in industry i . As a result the return on the ESGF will be lower than the return a capitalist can obtain from a portfolio composed of fund 1 and fund 2. Capitalists will then not invest into the ESGF and there will be no compliant firms in the economy. Thus a contradiction.
2. Taking into account consumers demand and firms demand given in (5) and (9), respectively, we can write the equilibrium condition on the goods markets:

$$\begin{cases} Y_1 = \frac{\gamma_1}{p_1} + \frac{p_2 Y_2}{p_1} a_{12} \\ Y_2 = \frac{\gamma_2}{p_2} + \frac{p_1 Y_1}{p_2} a_{21} \end{cases}$$

where $Y_i := Z_i/p_i$ is the aggregate production of good i and we claimed that consumer's aggregate wealth is equal to 1. The solution of this system provides expression (12).

- 2.-3. Note that in equilibrium the return of the three funds must be the same. Namely there is some $r > 0$ such that $r_1 = r_2 = r_F = r$. This because, first, if $r_1 \neq r_2$ then one of the two

sector would receive no capital, but this would lead to no production of one of the goods, and this cannot occur in equilibrium. Second the ESGF portfolio is obtained from investing in either one or both industries. Because in each industry firms are homogenous, it must be that $r_F = r$.

Observe that industry i 's aggregate profit is equal to $Z_i(1 - \alpha_{ij})$. A fraction $1 - \lambda$ of this profit will be distributed prorata to the capitalists. Hence, if the total amount of capital invested in industry i is K_i , then the return on investing in industry i must satisfy

$$r = \frac{Z_i(1 - \alpha_{ij})(1 - \lambda)}{K_i}$$

or equivalently

$$K_i = \frac{Z_i(1 - \alpha_{ij})(1 - \lambda)}{r}$$

Note that $Z_1(1 - \alpha_{12}) + Z_2(1 - \alpha_{21}) = \gamma_1 + \gamma_2 = 1$ and that the total amount of capital in the economy sum up to 1, i.e., $K_1 + K_2 = 1$. These equalities are satisfied only if $r = 1 - \lambda$ and $K_i = Z_i(1 - \alpha_{ij})$.

4. Take a firm in industry i , then its profit is equal to $\pi_i = \frac{p_i Y_i(1 - \alpha_{ij})}{K_i} = 1$, where the second equality follows from the fact that $p_i Y_i = Z_i$.
5. We already know from 2. that by investing his unit of capital, each capitalist gets $r = 1 - \lambda$. A typical entrepreneur's revenue is $\lambda \pi_i = \lambda$ because of 4. Consumers' aggregate wealth equals $\int_0^1 \lambda de + \int_0^1 1 - \lambda dc = 1$.
3. From point 4. we know that a firm equilibrium profit π_i equals 1. Using expression (10) we can write

$$\begin{cases} \frac{p_1}{p_2^{\alpha_{12}}} e_1^{\beta_1} = \frac{1}{\alpha_{12}^{\alpha_{12}}} \frac{1}{(1 - \alpha_{12})^{(1 - \alpha_{12})}} \\ \frac{p_2}{p_1^{\alpha_{21}}} e_2^{\beta_2} = \frac{1}{\alpha_{21}^{\alpha_{21}}} \frac{1}{(1 - \alpha_{21})^{(1 - \alpha_{21})}} \end{cases}$$

implying,

$$\begin{cases} p_1^{1 - \alpha_{12}\alpha_{21}} e_1^{\beta_1} e_2^{\alpha_{12}\beta_2} = \frac{1}{A_1} \\ p_2^{1 - \alpha_{21}\alpha_{12}} e_2^{\beta_2} e_1^{\alpha_{21}\beta_1} = \frac{1}{A_2} \end{cases}$$

where $A_i := \alpha_{ij}^{\alpha_{ij}}(1 - \alpha_{ij})^{(1-\alpha_{ij})}\alpha_{ji}^{\alpha_{ji}\alpha_{ij}}(1 - \alpha_{ji})^{(1-\alpha_{ji})\alpha_{ij}}$. Observe that

$$p_1^{-\gamma_1} p_2^{-\gamma_2} = \Theta e_1^{\beta_1 Z_1} e_2^{\beta_2 Z_2}$$

where $\Theta := A_1^{\frac{\gamma_1}{1-\alpha_{12}\alpha_{21}}} A_2^{\frac{\gamma_2}{1-\alpha_{12}\alpha_{21}}}$. Replacing this expression of the prices into (6), and considering that $E_i = e_i K_i$, we have that the equilibrium level of an individual utility is proportional to the following function of the levels of emissions

$$C \frac{e_1^{\beta_1 Z_1} e_2^{\beta_2 Z_2}}{(1 + e_1 K_1)^{\delta_1} (1 + e_2 K_2)^{\delta_2}} = U(e_1, e_2)$$

where $C := \Theta \gamma_1^{\gamma_1} \gamma_2^{\gamma_2}$.

■

Proof of Proposition 2. Observe that if one exogenously fix the level of emission in each industry, and let markets clear, the equilibrium allocation leads to the constrained social optimum given the level of emission. Consider a social planner who can fix the average emission level in each industry e_i, e_2 in order to maximize the individuals utility. Note that in equilibrium each individual's utility is proportional to $U(e_1, e_2)$. Taking the log of $U(e_1, e_2)$, and differentiating, one finds that the socially optimal level of emission in industry i is the minimum between 1 and the e_i solving the following first order condition:

$$\beta_i Z_i (1 + E_i) - \delta_i E_i = 0$$

The maximization problem has an internal solution only if $\delta_i < \frac{\beta_i(1+K_i)}{1-\alpha_{ij}}$. Otherwise it is optimal to let $e_i = 1$. ■

Proof of Lemma 1. Suppose that \hat{e}_i satisfy inequality (16). Note first that if $\eta_i = 1$ then $\hat{e}_i = 1$ and the ESGF induces no constraint on industry i firms. So the Lemma is trivially satisfied. Hence, take $\hat{e}_i < 1$. Consider a single entrepreneur in industry i who expects all other entrepreneurs in the same industry to comply by setting their individual emission level at \hat{e}_i . We have seen in the proof of Proposition 1 that conditionally on complying he will set $e_{i,f} = \hat{e}_i$. Second, let show that

choosing a non-compliant technology cannot be optimal. The best a non-compliant firm f can do is to maximize production by maximizing emission, $e_{i,f} = 1$. By choosing not to comply however, the entrepreneur takes the risk of not being financed. From the definition of $\Phi(e, \hat{e}_i)$ and expression (10), her expected payoff is

$$\left(\max\left\{0, \frac{1 - s_i}{1 - \eta_i s_i}\right\} \right) \lambda \left(p_i \left(\frac{\alpha_{ij}}{p_j} \right)^{\alpha_{ij}} \right)^{\frac{1}{1 - \alpha_{ij}}} (1 - \alpha_{ij}) \quad (28)$$

that is not larger than

$$\lambda \left(p_i \hat{e}_i^{\beta_i} \left(\frac{\alpha_{ij}}{p_j} \right)^{\alpha_{ij}} \right)^{\frac{1}{1 - \alpha_{ij}}} (1 - \alpha_{ij}) \quad (29)$$

only if S_i is not smaller than $S_i(\hat{e}_i)$. ■

Proof of Proposition 3.

The program of the ESGF is

$$\max_{e_1, e_2} \quad U(e_1, e_2) \quad (30)$$

$$s.t. \quad S_1(e_1) + S_2(e_2) \leq S \quad (31)$$

which we rewrite taking log of the objective function:

$$\max_{e_1, e_2} \quad \sum_{i=1,2} \beta_i Z_i \ln(e_i) - \delta_i \ln(1 + K_i e_i) \quad (32)$$

$$s.t. \quad \sum_{i=1,2} \frac{1 - e_i^{\frac{\beta_i}{1 - \alpha_{ij}}}}{1 - \eta_i e_i^{\frac{\beta_i}{1 - \alpha_{ij}}}} K_i \leq S \quad (33)$$

which we can see as:

$$\max_{e_1, e_2} \quad \sum_{i=1,2} f_i(e_i) \quad (34)$$

$$s.t. \quad \sum_{i=1,2} g_i(e_i) \leq S \quad (35)$$

1. First consider the case where S is small, preventing the fund to have high impact. Each e_i will

be in the neighborhood of 1. The ESGF should prioritize the sector for which $-f'_i(1)/g'_i(1)$ is the highest. Now, $f'_i(1) = \beta_i Z_i - \delta_i \frac{K_i}{1+K_i}$ and $g'_i(1) = -\frac{1}{1-\eta_i} \beta_i Z_i$. We use $K_i = (1-\alpha_{ij})Z_i$; we get the pecking-order rule: $i_0 = \operatorname{argmax}_{i \in \{1,2\}} (1-\eta_i) [\frac{\delta_i(1-\alpha_{ij})}{\beta_i(1+(1-\alpha_{ij})Z_i)} - 1]$. From $e_i^* = \frac{\beta_i}{(\delta_i - \beta_i Z_i)(1-\alpha_{ij})}$, it is easy to verify that $[\frac{\delta_i(1-\alpha_{ij})}{\beta_i(1+(1-\alpha_{ij})Z_i)} - 1] = \frac{1-e_i^*}{e_i^*(1+K_i)}$. Let $e_i(s_i) = S_i^{-1}(e_i) = \frac{K_i - s_i}{K_i - \eta_i s_i}$. Let \underline{S} such that $f'_{i_0}(e_{i_0}(\underline{S})) = f'_{-i_0}(1)$. Then investing all ESGF capital in industry i_0 is optimal as long as $S \leq \underline{S}$.

2. For $\underline{S} < S < S_1(e_1^*) + S_2(e_2^*)$, there is an interior solution that is determined by the system of equations $f'_i(e_i) = \xi g'_i(e_i)$, where ξ is the Lagrangian multiplier.
3. When the ESGF size $S > S_1(e_1^*) + S_2(e_2^*)$, the fund manages a capital that is large enough to induce the first best socially optimal behavior. That is, by investing at least $S_1(e_1^*)$ in industry i , for $i = 1, 2$, and fixing its policy $(e_1, e_2) = (e_1^*, e_2^*)$, the ESGF can guarantee that the average emission in each industry equals the socially optimal level.

■

Proof of Proposition 5. Suppose that the ESGF invests amounts $S_1 > 0$ and $S_2 > 0$ in industries 1 and 2, respectively. From the same argument used to show point 1 of Proposition 1, we have that in any given industry i , all firms comply. With internally consistent policies, $\hat{e}_j \leq \hat{e}_{U,i}$, implying all possible suppliers of a firm in industry i have emissions that are compatible with the firm indirect emission cap $\hat{e}_{U,i}$. Thus, in order to comply a firm in industry i just needs to have its direct emission not exceeding \hat{e}_i . Thus all firms in industry comply by setting $e_i = \hat{e}_i$ and we are back to the situation analyzed in section Section 4.2 where policies only focus on firms' direct emissions. ■

Proof of Proposition 6.

If the ESGF invests all its capital S in industry i , then $s_i = S$ and $s_j = 0$, and so $\left(\frac{K_i - s_i}{K_i - \eta_i s_i}\right) < 1$ and $\left(\frac{K_j - s_j}{K_j - \eta_j s_j}\right) = 1$. Suppose, all firms in industry i comply. Then there must be a non-nil fraction of firms in industry j that set their direct emission at $e_{j,f} \leq \hat{e}_{U,i}$, otherwise there would be no supplier eligible to sell to firms in industry i . Let us show that not all firms in industry j will set their emissions below $\hat{e}_{U,i}$. Suppose instead that each firm f in industry j sets $e_{j,f} \leq \hat{e}_{U,i}$. Then, an

entrepreneur in industry j would profit by setting up the only firm whose direct emission are $e_j = 1$ and then smuggling its product in the only market of good j . This will allow the firm to generate bigger profit compared to the other firms in the same industry. Because there is no ESG capital invested in industry j , by choosing a non-compliant technology the entrepreneur will not reduce the chance of being financed. Thus, a profitable deviation. Let show now that compliant firms in industry i , low emission firms in industry j , and high emission firms in industry j will set their direct emission at \hat{e}_i , \hat{e}_{U_i} , and 1, respectively. Suppose that compliant firms in industry i set direct emission at $e_i < \hat{e}_i$. Then an entrepreneur of this industry could set $e_{i,f} = \hat{e}_i$, without reducing the probability of being financed and being able to smuggle its production in the same market of the other firms in the industry, thus having a larger profit than other firms in the same industry. The same argument applies to low and high emission firms in industry j .

For both high emission and low emission firms co-existing in industry j , industry j entrepreneurs must be indifferent between high and low emission. Because $\left(\frac{1-s_j}{1-\eta_j s_j}\right) = 1$, emission level does not affect the probability of being financed. Hence we must have that high and low emission firms generate the same profit. That is,

$$\left(p_j(1) \left(\frac{\alpha_{ji}}{p_i(\hat{e}_i)}\right)^{\alpha_{ji}}\right)^{\frac{1}{1-\alpha_{ji}}} (1 - \alpha_{ji}) = \left(p_j(\hat{e}_{U_i}) \hat{e}_{U_i}^{\beta_j} \left(\frac{\alpha_{ji}}{p_i(\hat{e}_i)}\right)^{\alpha_{ji}}\right)^{\frac{1}{1-\alpha_{ji}}} (1 - \alpha_{ji}),$$

that is is possible only if $p_j(1) = \hat{e}_{U_i}^{\beta_j} p_j(\hat{e}_{U_i}) \leq p_j(\hat{e}_{U_i})$, where the inequality follows from $\hat{e}_{U_i} \leq 1$. We have thus shown 1 and 3.

Result 4, follows from the fact that consumers buy goods from the firms selling at the lowest prices, and $p_j(1) \leq p_j(\hat{e}_{U_i})$. Thus, they will buy good j only from high emission firms. Because all firms in industry i comply, these firms can only buy their input from low emission firms in industry j .

We can now write the equilibrium condition on good i market, and on good j markets for high

and low emission levels.

$$\begin{cases} Y_i = \frac{\gamma_i}{p_i(\hat{e}_i)} + \left(\frac{p_j(\hat{e}_{Ui})Y_{jL}}{p_i(\hat{e}_i)} + \frac{p_j(1)Y_{jH}}{p_i(\hat{e}_i)} \right) \alpha_{ji} \\ Y_{jL} = \frac{p_i(\hat{e}_i)Y_i}{p_j(\hat{e}_{Ui})} \alpha_{ij} \\ Y_{jH} = \frac{\gamma_j}{p_j(1)} \end{cases}$$

where, for industry j we used the subscript jL and jH to denote distinguish low emission and high emission variables, respectively. Solving this system one gets the following levels of sales revenues:

$$\begin{cases} Z_i = \frac{\gamma_i + \alpha_{ji}\gamma_j}{1 - \alpha_{12}\alpha_{21}} \\ Z_{jL} = \frac{\gamma_i + \alpha_{ji}\gamma_j}{1 - \alpha_{12}\alpha_{21}} \alpha_{ij} \\ Z_{jH} = \gamma_j \end{cases}$$

From these equilibrium level of sales, using the same argument as for the proof of point 2 in Proposition 1 we have that $K_i = Z_i(1 - \alpha_{ij})$, $K_{jL} = Z_{jL}(1 - \alpha_{ji})$ and $K_{jH} = Z_{jH}(1 - \alpha_{ji})$. Thus, property 2.

We can now show that if the level of $(\hat{e}_i, \hat{e}_{Ui})$ satisfies Condition 1 and firms in industry j behave as described above, then all firms in industry i comply. First, note that if all other firms in industry i comply, then the best a non-compliant firm in industry i can do is to set its direct emission at 1, buy good j at the low price, $p_j(1)$, from high emission firms, and smuggle its production for $p_i(\hat{e}_i)$. Thus complying is optimal if

$$\underbrace{\left(p_i(\hat{e}_i) \left(\frac{\alpha_{ij}}{p_j(1)} \right)^{\alpha_{ji}} \right)^{\frac{1}{1-\alpha_{ji}}}}_{\text{maximum profit for non-compliant firm}} \underbrace{\max \left\{ 0, \frac{K_i - S}{K_i - \eta_i S} \right\}}_{\text{probability of being financed}} \leq \underbrace{\left(p_i(\hat{e}_i) \hat{e}_i^{\beta_i} \left(\frac{\alpha_{ij}}{p_j(\hat{e}_{Ui})} \right)^{\alpha_{ij}} \right)^{\frac{1}{1-\alpha_{ij}}}}_{\text{profit of compliant firm}} (1 - \alpha_{ij}).$$

By replacing $p_j(1)$ with $(\hat{e}_{Ui})^{\beta_j} p_j(\hat{e}_{Ui})$ and simplifying one gets Condition 1.

The level of an individual's utility is given by equation (6). Consumers purchase good j at price $p_j(1)$, thus an individual with wealth w achieves:

$$\frac{\left(\frac{\gamma_i}{p_i(\hat{e}_i)} \right)^{\gamma_i} \left(\frac{\gamma_j}{p_j(1)} \right)^{\gamma_j}}{(1 + E_1)^{\delta_1} (1 + E_2)^{\delta_2}} w \quad (36)$$

Considering that $\pi_i = \pi_{jL} = 1$ and following similar steps as for the proof of Proposition 1, we have

$$p_i(\hat{e}_i)^{-\gamma_i} p_j(\hat{e}_{U_i})^{-\gamma_j} = \Theta \hat{e}_i^{\beta_i Z_i} \hat{e}_{U_i}^{\beta_j Z_j}$$

where $\Theta = A_1^{\frac{\gamma_1}{1-\alpha_{12}\alpha_{21}}} A_2^{\frac{\gamma_2}{1-\alpha_{12}\alpha_{21}}}$ and $A_i = \alpha_{ij}^{\alpha_{ij}} (1 - \alpha_{ij})^{(1-\alpha_{ij})} \alpha_{ji}^{\alpha_{ji}\alpha_{ij}} (1 - \alpha_{ji})^{(1-\alpha_{ji})\alpha_{ij}}$. Recalling from point 3 above that $p_j(1) = \hat{e}_{U_i}^{\beta_j} p_j(\hat{e}_{U_i})$, we can write

$$p_i(\hat{e}_i)^{-\gamma_i} p_j(1)^{-\gamma_j} = \Theta \hat{e}_i^{\beta_i Z_i} \hat{e}_{U_i}^{\beta_j (Z_j - \gamma_j)}$$

Observing that $Z_j - \gamma_j = \alpha_{ij} Z_i$, and the average emission in industry j is $e_j = (\theta_j + (1 - \theta_j)\hat{e}_{U_i})$ we can express the individual utility (36) as

$$C \frac{e_i^{\beta_i Z_i} \left(\frac{e_j - \theta_j}{1 - \theta_j} \right)^{\beta_j \alpha_{ij} Z_i}}{(1 + \hat{e}_i K_i)^{\delta_1} (1 + e_j K_j)^{\delta_2}} w = U_I(e_i, e_j) w.$$

■

Proof of Lemma 3. The proof is in two steps. First we show that if small size ESG is willing to reduce the average emission on a given industry, say industry 2, the most effective way is to put all its capital in the friction industry, i.e. the industry with the smallest η_i . Second, given that maximum impact is achieved by investing all capital in the friction industry, we show that the ESGF should focus on reducing the emission of the critical industry, i.e. the one with the smallest e_i^* .

Step 1: Suppose ESGF wants to reduce emission in industry 2. By investing all its capital in this industry it can bring its direct emissions to

$$e_{2,Dir}(S) := \left(\frac{K_2 - S}{K_2 - \eta_2 S} \right)^{\frac{1-\alpha_{21}}{\beta_2}}$$

If instead it invests all its capital in industry 1, imposes no cap on industry 1's direct emission, to

focus on industry 1's indirect emission, it can bring industry 2 emission down to:

$$e_{2,Ind}(S) := \theta_2 + (1 - \theta_2) \left(\frac{K_1 - S}{K_1 - \eta_1 S} \right)^{\frac{1 - \alpha_{12}}{\alpha_{12}\beta_2}}$$

Note that $e_{2,Dir}(0) = e_{2,Ind}(0) = 1$. With some algebra we get

$$\left. \frac{\partial e_{2,Dir}}{\partial S} \right|_{S=0} = -\frac{1 - \eta_2}{\beta_2 Z_2}$$

and

$$\left. \frac{\partial e_{2,Ind}}{\partial S} \right|_{S=0} = -\frac{1 - \eta_1}{\beta_2 Z_2}.$$

Hence, the marginal impact of ESGF capital in reducing e_2 is stronger when investing in the industry with the smaller η_i . The same argument applies if the ESGF wanted to reduce industry 1's emission.

Step 2: Without loss of generality let assume that $\eta_1 < \eta_2$, implying the the fund must invest all its capital in industry 1. For S small the marginal gain in (the log of) social welfare from using the capital to reducing industry 1 or industry 2 emission is

$$\left. \frac{\partial U(e_1, e_2)}{\partial e_1} \frac{\partial e_{1,Dir}}{\partial S} \right|_{S=0} = - \left(\beta_1 Z_1 - \frac{\delta_1 K_1}{1 + K_1} \right) \frac{1 - \eta_1}{\beta_1 Z_1} = \frac{1 - e_1^*}{e_1^*(1 + K_1)} (1 - \eta_1)$$

$$\left. \frac{\partial U_I(e_1, e_2)}{\partial e_2} \frac{\partial e_{2,Ind}}{\partial S} \right|_{S=0} = - \left(\beta_2 Z_2 - \frac{\delta_2 K_2}{1 + K_2} \right) \frac{1 - \eta_1}{\beta_2 Z_2} = \frac{1 - e_2^*}{e_2^*(1 + K_2)} (1 - \eta_1),$$

respectively. Where, we used $e_1 = e_2 = 1$ for $S = 0$ and we have replaced $\left(\beta_i Z_i - \frac{\delta_i K_i}{1 + K_i} \right)$ with $\frac{\beta_i Z_i (e_i^* - 1)}{e_i^* (1 + K_i)}$, $i = 1, 2$, using the definition of e_i^* . ■

Proof of Proposition 7. Let us say that a ESGF policy is “direct” if it caps exclusively firms direct emissions. We say the policy is “indirect” if the ESGF invests all S is in industry i and imposes to this industry direct and/or indirect emissions caps.¹³ To fix idea we set $i = 1$ and $j = 2$. The argument of the proof unfolds as follows. Fix an arbitrary level of $e_1 \in [0, 1]$ and let $S < S^*$. Consider the minimum level of e_2 that an ESGF of size S can impose to industry 2

¹³If $S > K_i$, the remaining ESGF's capital is invested in industry j without requiring any emission cap to this industry.

while imposing e_1 to industry 1. Namely let $e_{2D}(e_1, S)$ and $e_{2I}(e_1, S)$ be these levels for the direct and the indirect policy, respectively. We show that for the value of parameters in the proposition $e_{2D}(e_1, S) \geq e_{2I}(e_1, S)$ for all feasible e_1 . Hence the set of (e_1, e_2) that can be implemented with the direct policy is included in the set of (e_1, e_2) that can be implemented with the indirect policy. Thus, the impact of an appropriate direct policy is greater than the impact of the best of direct policies.

For the direct policy, rearranging $S_1(e_1) + S_2(e_2) \leq S$, we have that

$$e_{2D}(e_1, S) = \max \left\{ 0, \frac{K_2 - (S - S_1(e_1))}{K_2 - \eta_2(S - S_1(e_1))} \right\}^{\frac{1-\alpha_{21}}{\beta_2}}.$$

For the indirect policy, rearranging (22) we have that

$$e_{2I}(e_1, S) = \min \left\{ 1, \theta_2 + (1 - \theta_2) e_1^{-\frac{\beta_1}{\alpha_{12}\beta_2}} \max \left\{ 0, \frac{K_1 - S}{K_1 - \eta_1 S} \right\}^{\frac{1-\alpha_{12}}{\alpha_{12}\beta_j}} \right\}$$

Observe that for $e_1 \leq \max\{0, \frac{K_1 - S}{K_1 - \eta_1 S}\}^{\frac{1-\alpha_{12}}{\beta_1}}$, one has $e_{2D}(e_1, S) = e_{2I}(e_1, S) = 1$. Hence let consider $e_1 > \max\{0, \frac{K_1 - S}{K_1 - \eta_1 S}\}^{\frac{1-\alpha_{12}}{\beta_1}}$. One has $e_{2D}(e_1, S) < 1$ and $e_{2I}(e_1, S) < 1$. Fix $\eta_1 < 1$ and let $\eta_2 = 1 - \varepsilon > \eta_1$, then

$$\lim_{\varepsilon \rightarrow 0} e_{2D}(e_1, S) = 1 > e_{2I}(e_1, S)$$

hence, for any S there is $\eta_2 \in (\eta_1, 1)$ such that $e_{2D}(e_1, S) > e_{2I}(e_1, S)$, for all $e_1 > \max\{0, \frac{K_1 - S}{K_1 - \eta_1 S}\}^{\frac{1-\alpha_{12}}{\beta_1}}$.

Observe that because $0 < \gamma_2, \alpha_{1,2} < 1$, if $\alpha_{1,2} - \gamma_2$ is large, then there is some ε small such that $\gamma_2 \leq \varepsilon$ and $\alpha_{1,2} \geq 1 - \varepsilon$. Note that because $\alpha_{1,2} > 1 - \varepsilon$, one has that $\lim_{\varepsilon \rightarrow 0} S_1(e_1) = K_1$ and $\lim_{\varepsilon \rightarrow 0} K_1 = 0$. Hence,

$$\lim_{\varepsilon \rightarrow 0} e_{2D}(e_1, S) = \left(\frac{K_2 - S}{K_2 - \eta_2 S} \right)^{\frac{1-\alpha_{21}}{\beta_2}} > 0$$

where the inequality follows from the fact that $S < S_1(e_1^*) + S_2(e_2^*)$, $\lim_{\varepsilon \rightarrow 0} S_1(e_1^*) = 0$ and $S_2(e_2^*) <$

K_2 . Also

$$\lim_{\varepsilon \rightarrow 0} \max \left\{ 0, \frac{K_1 - S}{K_1 - \eta_1 S} \right\}^{\frac{1-\alpha_{12}}{\alpha_{12}\beta_j}} = 0$$

Consider now $e_{2I}(e_1, S)$. Because $\gamma_2 < \varepsilon$ one has that $\lim_{\varepsilon \rightarrow 0} \theta_2 = 0$. Thus $\lim_{\varepsilon \rightarrow 0} e_{2I}(e_1, S) = 0$.

By continuity there is $\varepsilon > 0$ small enough such that $e_{2I}(e_1, S) < e_{2D}(e_1, S)$ ■

Proof of Proposition 8.

1. For ϕ large enough, ESGF return will too small to attract any capitalist.
2. Note first that $S = 0$ is always an equilibrium: if capitalists expect no impact they do not invest in the ESGF can not have impact. To see that S can be positive, note that equality (26) can be restated as

$$I = \phi \frac{(1-\lambda)\hat{U}(S)}{1-S}$$

Considering that I is given by (24), we need to show that for ϕ small enough there is $S > 0$ such that

$$\hat{U}(S) - U(1, 1) = \phi \frac{(1-\lambda)\hat{U}(S)}{1-S} \quad (37)$$

Note that the r.h.s. is strictly positive for all S and goes to plus infinity as S goes to 1.

Thus for $S = 0$ and $S = 1$ the r.h.s is strictly larger the l.h.s. Fix any $\varepsilon \in (\lambda, 1)$ and let $\phi \in \left(0, \frac{(1-\varepsilon)(\hat{U}(\varepsilon)-U(1,1))}{(1-\lambda)\hat{U}(\varepsilon)}\right)$. Then for $S = \varepsilon$ the r.h.s. of (37) is strictly smaller than the l.h.s.

Thus (27) has at least two distinct strictly positive solutions.

3. The first best social optimum is achieved if $S > S^*$. Note that if $\phi = \phi^*$ then equation (37) has a solution for $S = S^*$. For all $\phi < \phi^*$ it has a solution for $S > S^*$.

■

Micro-fundation of the matching model (For online publication)

We consider the following matching mechanism between entrepreneurs and capital. Fix industry i . In equilibrium K_i is the total amount of capital invested the industry and is also equal to the equilibrium mass of entrepreneurs in the industry. Let s_i be the fraction of this capital coming from ESGF. We can see the matching as the result of the following dynamic process. In every period t entrepreneur that have not been yet finance and capital that has not been invested yet are randomly matched. Entrepreneur who are not financed in t come back to the matching market in $t+1$. Capital that has not be invested in t is returned to investors. A fraction s_i of this capital is reinvested via ESGF, the remaining $1 - s_i$ is invested via non-ESG funds. We denote with $1 - \eta_i \in [0, 1]$ the hazard rate that the capital matching process stops before the entrepreneur finds an appropriate capital provider.

Lemma 4 *In every period t , an entrepreneur searching for capital is financed with probability 1 if it complies and with probability $1 - s_i$ if she does not comply.*

Proof. Let denote with C_t the total amount of capital not yet matched at time t . The fraction of this capital that is managed by ESGF is s_i . Let M_t denote the mass of entrepreneurs that at beginning of time t has not been matched. Let n_t be the fraction of these entrepreneur that are not compliant. All compliant entrepreneurs can be financed with ESG and non-ESG funds, thus the first result. We first show by induction that for all t $M_t = C_t = s_i$. For $t = 0$ we know that in equilibrium the total amount of capital invested in industry i and the total mass of entrepreneur in industry i are equal to K_i , thus $C_0 = M_0 = K_i$. At the end of the first period the mass of capital not invested C_1 is equal to the ESG capital that was matched with non-complying entrepreneurs, and is $C_1 = s_i C_0 n_0$. The mass of entrepreneur not financed equals to the non-complying entrepreneurs matched with ESG capital and equals $M_1 = n_0 M_0 s_i = C_1$. Note that the end of the first round all compliant entrepreneur have been financed, thus if $n_0 = 0$, then for all $t > 0$, $M_t = C_t = n_t = 0$ and the mayching process only lasts one round. If $n_0 > 0$ then fir all $t > 0$, we have $n_t = 1$. In this case we can apply the same argument for a general period t to have that if $M_t = C_t$, and $M_{t+1} = C_{t+1} = s_i C_t$. In period t a non-compliant entrepreneur is financed only if she is matched

with non-ESG capital and this occurs with probability $\frac{C_t - s_i C_t}{M_t} = 1 - s_i$ because $C_t = M_t$. ■

Consider a compliant entrepreneur. She will be funded in the first period and exit the matching market. Her revenue will be $V_{ic} = \pi_{i,c}\lambda$, where $\pi_{i,c}$ is the profit of compliant firm in industry i . Non-compliant entrepreneurs can only be financed with non-ESG capital. From the above lemma, as long as a non-complying entrepreneur has not been financed, her expected payoff is

$$V_{in} = (1 - s_i)\lambda\pi_{i,n} + s_i\eta_i V_{in}$$

That is, with probability $1 - s_i$, the entrepreneur is matched with non-ESG capital. She starts the firm immediately and retains a fraction λ of a non-compliant firm profit $\pi_{i,n}$. With probability s_i the entrepreneur faces a ESG capital provider, she is not financed and has to go back to the matching market that will provide with a new capital provider with probability η_i . This implies

$$V_{in} = \lambda\pi_{i,n} \left(\frac{1 - s_i}{1 - \eta_i s_i} \right).$$

The entrepreneur will choose a compliant technology only if $V_{in} \leq V_{ic}$, that is

$$\underbrace{\pi_{i,n} \left(\frac{1 - s_i}{1 - \eta_i s_i} \right)}_{\phi(1, \hat{e}_i)} \leq \pi_{i,c}$$

That corresponds to condition (15). Note that in equilibrium the ESGF choose its portfolio and the emission caps as to induce all entrepreneurs to comply and are all matched in the very first round of the our search game, that hence only lasts one period.