Trade-offs in the Design of Fair Value Standards*

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Abstract: This paper analyzes the information qualities of the current fair value standard in US-GAAP and IFRS in terms of value relevance and faithful representation. The current standard favors reliable market-based inputs over sometimes more relevant entity-specific inputs as it requires preparers to use the measurement approach, which maximizes observable inputs and minimizes unobservable inputs. We apply a model of rational expectations in line with Fischer and Verrecchia (2000) and introduce an information structure including public market-wide signals and managerial private signals. In contrast to common intuition, we find that maximizing observable inputs does not necessarily lead to a better faithful representation in the sense of a lower managerial bias. In fact, an alternative standard, which requires the maximization of unobservable inputs, can simultaneously provide a lower managerial bias and a higher value relevance than the current standard, depending on the information environment. This is the case, for example, when the corporate governance system is moderate and the uncertainty about the underlying asset is sufficiently high. The results are important for regulators and scholars since they show that the trade-offs for standard setters are not as clear-cut as commonly suggested.

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1 Introduction

"An entity shall use valuation techniques [...] to measure fair value, maximizing the use of relevant observable inputs and minimizing the use of unobservable inputs." (IFRS 13.61) (IASB, 2017).

FASB as well as the IASB agree on the approach regarding fair value measurements. The common convergence project of the two standard setting boards documents this agreement in the international standards IFRS 13 and in US-GAAP ASC 820. Amongst other definitions and guidances, the standards require a hierarchy of fair value inputs. If available, the preparer should use quoted prices on active markets (Level 1-inputs), inputs other than quoted prices that are observable (Level 2-inputs), and unobservable inputs possibly based on the manager's own evaluation models (Level 3-inputs) in descending order. The focus of the standard on observable inputs from Level 1 and 2 aims to constrain the manager's possibility to inflate accounting earnings. Market-based observable inputs strengthen the market's confidence in the financial statements due to its higher traceability and reliability. On the contrary, unobservable inputs might be easier to manipulate but provide investors with new information, which is private to the firm, while observable inputs are public knowledge. Hence, when facing illiquid assets from Level 2- or Level 3-inputs, scholars and practitioners often refer to the classic trade-off between relevant and reliable information (Laux and Leuz, 2009).

However, the properties of the inputs to measuring fair values are not sufficient to analyze whether a standard serves the intended purpose. According to the conceptual framework, the objective of US-GAAP and IFRS is to provide information that is useful for investors. Information is useful if it meets the qualitative characteristics.

These are relevance, which means the information is capable of making a difference in the decisions made by users, and faithful representation (formerly reliability), which means information is complete, neutral, and free from error (FASB, 2018; IASB, 2018). We open the black box of the financial statement process and analyze how the properties of the current standard translate into the qualitative characteristics of accounting standards. Based on this analysis, we ask the question whether an alternative standard exists, which meets the qualitative characteristics better than the current standard.

Our model shows that reliable inputs do not always lead to a better faithful representation and more relevant inputs do not always lead to higher value relevance of information. We derive our results with a rational expectation model based on Fischer and Verrecchia (2000). A manager and the market both receive two public signals about the underlying value of a firm, which is measured at fair value. In addition, the manager receives two private signal corresponding to each public signal. Then, the manager transmits a possibly biased report about the firm's value to the market, which forms a price based on the report and the two public signals.

For example, a firm wants to determine the fair value of a building and it has two measurement options available: the market approach and the income approach. The market approach measures the fair value of the own building based on a recent sale of a similar building and adjusts for differences, such as location or conditions of the facilities, by using multiples. The public signal could be the selling price of the similar building, the private signal could be the manager's inside knowledge about the characteristics of the own building and the necessary adjustments. The income approach measures the fair value as the discounted cash flow of future expected rent income. The public signal could be the average rent in the neighborhood, the private signal could be the manager's insights about the appropriate discount factor. Both the market or income approach could be the one with more observable inputs and therefore be required by the current standard.

We first analyze how a change in the information structure influences the qualitative characteristics of each measurement approach. We measure value relevance in line with Fischer and Verrecchia (2000) as the added precision about the underlying firm value by the audited report. Our measure for faithful representation is the manager's bias. The manager's incentive to bias depends on her privately observed participation in the market price of the firm and her expected costs of biasing, which increase when the report deviates from the accounting standard's requirement.

In this context, we find, for example, that an increase in the uncertainty about the fundamental value can increase or decrease the expected bias depending on the strength of the corporate governance system. Higher uncertainty directly increases the market's need for information, which is why it pays more attention on the manager's report and biasing becomes more attractive. However, the manager also reacts by increasing the weight on private and public information contained in the report. When the corporate governance regime is weak, the uncertainty about the manager's degree of biasing is high. Then, the market's demand for precise information is not satisfied, yet, and it further increases the attention on the manager's report to utilize the manager's private information. In contrast, when the corporate governance system is strong, the uncertainty about the manager's degree of biasing is low. Then, the market perceives the manager's increase of private and public information as overcorrection and de-emphasizes the attention to the manager's report. For high values of uncertainty about the fundamental value, this negative indirect effect can prevail and the bias decreases. In contrast, value relevance always increases.

The results apply for both measurement approaches, market or income approach. However, typically both approaches have different features in terms of public and private information and also lead to a different valuation. Due to the complex manager-investor interaction, we then ask the question whether an alternative standard, which requires preparers to maximize the use of unobservable instead of observable inputs, could sometimes be beneficial to investors. We find, for example, that the alternative standard delivers both a lower managerial bias and a higher value relevance than the current standard, when the corporate governance system is moderate and the uncertainty about the underlying value of the item is sufficiently high. The insights are important to scholars and standard setters as they point out that the trade-off between qualitative characteristics of decision useful accounting standards is not as clear cut as commonly suggested.

We further extend our analyses to the hypothetical case when the standard requires the preparer to use information from both approaches to report a single number in the financial statement in the sense of an average. We show that also this approach is not always dominant in terms of the qualitative characteristics. For example, when the manager's private information with respect to the market approach is very noisy, the market benefits when this information is disregarded and the fair value is reported under the income approach only.

Lastly, we consider the influence of a valuation specialist used by an auditor to verify the accuracy of the reported numbers. Thereby, the auditor determines a budget for the specialist, where a higher budget increases the precision of her private information. A stricter liability regime increases the auditor's incentive to generate a more precise audit signal. We find that in low to moderate auditor liability regimes, such as Germany or other continental European countries, value relevance of Level

3-assets is higher than Level 2-assets. In contrast, in strict liability regimes, such as the US, Level 2-assets are more value relevant. Simultaneously, the expected bias is larger for Level 2-assets in most jurisdiction, except when the liability is extremely lenient.

Our paper relates to Fischer and Verrecchia (2000), who provide the basic framework for our model. They study a manager with uncertain reporting objective and a capital market, which values the firm based on the manager's report. We share some common comparative static results, such as an increase in the marginal cost of biasing decreases the expected bias and increases value relevance. However, our distinction between public and private information generates additional insights, for example, that an increase in the uncertainty about the fundamental value can increase or decrease the expected bias. Further, we compare the qualitative characteristics of two measurement alternatives and show the influence of changes in the information environment on their rank order. We find that more observable inputs do not necessarily lead to more reliable financial statements.

In our extension, we add a valuation specialist on the side of the auditor. Similar to our model Caskey et al. (2010) and Patterson et al. (2019) base their studies on Fischer and Verrecchia (2000) and include an additional party verifying the manager's reporting. Caskey et al. (2010) consider an audit committee, which is responsible for the final report using the manager's report as an input. Similar to the auditor in our model, the audit committee has private information from a diligence process. Patterson et al. (2019) add an auditor.[1] We share with these studies that higher

[1] Other studies relate the auditor's liability to accounting characteristics. Kronenberger and Laux (2021) show that a higher threat of litigation does not always increase conservatism, as commonly suggested. The reason is that also the auditor reacts to higher litigation and increases audit effort. Therefore, the firm optimally chooses less conservatism when the cost of auditing is low. Other studies analyze the impact of liability on accounting manipulation or financial reporting quality

auditor liability increase the price informativeness. We add to this literature by connecting the auditor's liability to the measurement approaches in the current fair value standard. We find that the alternative standard that maximizes unobservable inputs dominates the current standard when the auditor's liability is moderately high.

Fair value, in general, is subject to a lively debate amongst practitioners and researchers. The discussion focuses on two main aspects: The contribution of fair value to the financial crisis (Barth, 2004; Kashyap and Stein, 2004; Laux and Leuz, 2009), and the usefulness of fair value in relation to historic cost accounting (Allen and Carletti, 2008; Plantin et al., 2008; Laux and Leuz, 2009). Beyond this discussion, the fair value hierarchy on inputs is especially relevant to this paper. Plantin and Tirole (2018) and Mahieux (2021) show the impact of public information (more Level 2-inputs) on financial stability. Plantin and Tirole (2018) find that excessive use of information generated by other firms' asset sales and insufficient use of the realization of a firm's own capital gains reduces market liquidity and the informativeness of price signals. Mahieux (2021) shows that more public information increases the systemic risk. Empirical literature finds both a higher value relevance of Level-2 inputs (Song et al., 2010; Riedl and Serafeim, 2011) and a higher value relevance of Level 3-inputs (Altamuro and Zhang, 2013; Lawrence et al., 2016; Chung et al., 2017). Our theoretical framework specifies the conditions in more detail and provides a robust guidance for future hypothesis development.

(Hillegeist and Stein (1999), Newman et al. (2005) or Ewert and Wagenhofer (2019)). An influence of liability on the firm's internal controls is studied, for example, in Nelson et al. (1988) or Pae and Yoo (2001).

2 Model Description

Consider a one-period reporting game with two risk-neutral players: the manager of a firm and a rational financial market. The firm owns a single asset which it intends to measure at fair value and whose unknown value is v. The prior beliefs about v of all players are normally distributed with mean μ_v and variance σ_v^2.

Information Endowment. There are two public signals about the firm value available for all market participants, $x = v + \epsilon_x$ and $y = v + \epsilon_y$, where ϵ_x and ϵ_y are uncorrelated noise terms with $\epsilon_x \sim N(\mu_{\epsilon_x}, \sigma_{\epsilon_x}^2)$ and $\epsilon_y \sim N(\mu_{\epsilon_y}, \sigma_{\epsilon_y}^2)$. To illustrate the connection of the firm value and the public information, suppose that the firm holds an apartment building as the only asset and its earnings are generated based on rent income. Then, public information x could be the price of a similar building from a recent sale and ϵ_x the information noise created by the adjustments necessary to reflect the economic conditions of the firm's own building, such as a different year of construction, location or number of floors. Fair value determination based on information x follows the market approach in accordance with IFRS 13 and ASC 820 (IASB, 2017; FASB, 2011). Public information y could be valuation of the average rent income in the entire neighborhood and ϵ_y represents the information noise to projected future rents from adjustments due to a changing demand structure in the future or from uncertainty about the appropriate discount rates. Fair value determination based on information y follows the income approach in accordance with IFRS 13 and ASC 820. Both public signals, x and y, can be considered as Level 2-inputs.

In addition to the public signal, the manager of the firm receives a private signal about the information noise of each public signal, $s_x = \epsilon_x + \eta_x$ and $s_y = \epsilon_y + \eta_y$

with uncorrelated noise terms $\eta_x \sim N(0, \sigma^2_{\eta x})$ and $\eta_y \sim N(0, \sigma^2_{\eta y})$. This private signal simply expresses the manager's superior knowledge about the necessary adjustments of the public information to evaluate the firm's building. Both private signals of the manager, s_x and s_y, can be considered as Level 3-inputs. The study does not consider the use of Level 1-inputs.

Timeline. The game has three dates. At date 1, the firm value v is realized but unknown to all parties. At date 2, the manager privately observes signals s_x and s_y and generates a report, r. At date 3, the market evaluates the firm value based on the public information, x and y and the manager's report, r, which leads to a market price of P.

The Manager's Incentives. The manager participates in the firm's market value, P. The manager's personal emphasis on the market price, θ, is private knowledge to the manager, while the market has priors about θ, which follow a normal distribution with mean μ_θ and variance σ^2_θ. This assumption is in line with Fischer and Verrecchia (2000) and Caskey et al. (2010). Although CEO compensation is mostly transparent, the market can never fully understand the entire extent of the manager's motivation, which also includes indirect facets such as career concerns or reputation as well as personal goals. The manager has quadratic cost from a reporting bias whenever the manager's report r deviates from the fair value standard, $E[v|n, s_n]$ with $n \in \{x, y\}$.[2] When the standard requires the market approach, then the preparer uses the public information $n = x$ and the private information $s_n = s_x$; when the standard requires the income approach, then the preparer uses the public information $n = y$ and the private information $s_n = s_y$. The measurement approach not required by the standard

[2]Theoretically, the manager could also bias by applying the wrong measurement approach. We do not address this form of biasing.

is described by $m \neq n$, with $n \in \{x, y\}$. In general, both approaches could be the required approach that delivers more observable and less unobservable inputs, depending on the asset and the market conditions. We use the general form (using notation n and m) in section 3 and assume a specific information environment (using notation x and y) in section 4 and 5 to compare the currently required fair value standard with the alternative standard. Parameter c reflects the marginal cost of biasing to the manager. Higher cost indicate a stronger reporting oversight in line with Caskey and Laux (2016).[3] In sum, the manager's utility is given by

$$U_M = \theta P_n - c(r_n - E[v|n, s_n])^2. \tag{1}$$

3 Measuring Fair Value with observable and unobservable inputs

3.1 Equilibrium

In this section, we establish the general equilibrium of the manager-market reporting game. In accordance with Fischer and Verrecchia (2000), we focus on linear strategies. Thus, the reported value of the asset required by the standard is the conditional expected value weighing the respective observable and unobservable inputs.

$$E\left[v|n, s_n\right] = \beta_{n,0} + \beta_n n + \beta_{s_n} s_n. \tag{2}$$

[3]Only the manager receives the private signal, s_n, and knows $E[v|n, s_n]$. Thus, we assume that $E[v|n, s_n]$ is unveiled internally ex-post. Parameter c could also reflect a misreporting penalty as in Caskey et al. (2010). Then, $E[v|n, s_n]$ is unveiled ex-post by a court.

$\beta_{n,0}$ represents the intercept, β_n represents the required weight on the public information, and β_{s_n} the required weight on the private information, where $n = x$ when the manager should use the market approach or $n = y$ for the income approach.

Then, the manager maximizes her expected utility by reporting r given the information available:[4]

$$r_n^* = \arg\max_{r_n} U_M = \arg\max_{r_n} E[\theta \cdot \widehat{P}_n - c(r_n - E[v|n, s_n])^2 | n, m, s_n, s_m, \theta] \tag{3}$$

The manager observes both public signals, n and m, the corresponding private signals, s_n and s_m, and the realization of the participation in the market price, θ. Further, he conjectures the market's price evaluation. The market evaluates the price given its available information as

$$P_n = E\left[v|n, m, r_n\right] \tag{4}$$

The market observes both public signals as well as the manager's report. However, it does not know the manager's underlying strategy since θ is privately observed. Based on the manager's first order-condition of (3) and the market's price reaction, we determine the following unique linear equilibrium, where all conjectures are true.

Proposition 1 *There is a unique linear equilibrium for the manager-market reporting game.*

The manager's reporting strategy is: $r_n^* = E\left[v|n, s_n\right] + \frac{\theta \cdot \psi_{r_n}^*}{2c}$.

The market's pricing strategy is: $P_n^* = \psi_{0,n}^* + \psi_n^* n + \psi_m^* m + \psi_{r_n}^* r_n$, *where*

$\psi_{n,0}^* = \mu_v - \psi_m^* \mu_m - \psi_n^* \mu_n - \psi_{r_n}^* \mu_{r_n}$, $\psi_n^* = A_n/B$, $\psi_m^* = A_m/B$, $\psi_{r_n}^* = A_{r_n}/B$

[4]Conjectures are market with "ˆ"

$A_i(j,k) = \sigma_v/\sigma_i \left(\rho_{vi} + \rho_{jk}\left(\rho_{jv}\rho_{ki} + \rho_{ji}\rho_{kv}\right) - \rho_{ji}\rho_{jv} - \rho_{ki}\rho_{kv} - \rho_{vi}\rho_{kj}^2\right)$

$B = 1 - \rho_{mr_n}^2 - \rho_{nr_n}^2 - \rho_{nm}^2 + 2\rho_{mr_n}\rho_{nr_n}\rho_{nm}$ *with* $i,j,k \in \{r_n, n, m\}$.

Proofs are in the Appendix.

The manager's optimal reporting strategy consists of two parts: The expected value of the firm according to the required measurement approach, $E[v|n,s_n]$, and the managerial bias, $\frac{\theta\cdot\psi_{r_n}^*}{2c}$. Without incentives to bias, the manager would simply replicate what the standard requires. Thus, the manager's weights on the public information n or m, the private information s_n or s_m, as well as the intercept are identical to the expectation according to the standard from (2).[5] Due to the manager's incentives, which originate from the participation in the market price, the manager adds a bias. The bias equals the manager's participation in the price, θ, times the market's reaction to the manager's report, $\psi_{r_n}^*$, often referred to as earnings response coefficient (ERC), over twice the cost of manipulation, c.

3.2 Comparative Statics

The underlying idea for the design of the standard is that observable inputs limit managerial discretion and help to provide useful, reliable information even if it is generated at the expense of information from unobservable inputs, which might provide investors with new information. However, the qualities of the inputs are not sufficient to evaluate whether each measurement approaches delivers the intended outputs. Therefore, we analyze in this section the effect of the information environment on the qualitative characteristics for decision useful information. These qualitative

[5] This also implies that the manager does not use the information of the alternative approach, m and s_m.

characteristics are - according to the conceptual framework - faithful representation, proxied by the manager's expected bias, and value relevance, proxied by the price efficiency of the report.

The manager's expected bias. We define the manager's expected bias as the expected difference between the manager's report and the standard's requirement

$$Eb_n = \frac{\mu_\theta \cdot \psi_{r_n}}{2c}. \tag{5}$$

Hence, with the exception of μ_θ and c, the influence of any information component on the managerial bias is determined by changes in the market's reaction to the report, ψ_{r_n}.[6]

Value Relevance. We define value relevance in accordance with Fischer and Verrecchia (2000) as the price efficiency of the final report. They describe price efficiency as the remaining uncertainty after the report is issued. We use a slightly different specification in that we construct the measure as the precision added by the report in order to receive an increasing notion of value relevance. Thus, our measure is simply the reciprocal of value relevance in Fischer and Verrecchia (2000).[7] Without the report, the market has public information n and m available, which yields a precision of the firm value of $1/Var(v|n, m)$. With the report, it changes to $1/Var(v|n, m, r_n^*)$. Hence, value relevance is given by

[6] We could also specify faithful representation as the realized bias from the market's perspective, which equals $\frac{\theta \cdot \psi_{R_n}}{2c}$. Since the realized bias differs only in θ from the expected bias, the comparative statics results are almost identical.

[7] Referring to the ERC ψ_{r_n} as value relevance in a setting with multiple public information sources would be incomplete. The reason is that a change in the information environment also changes the market's strategy in equilibrium with respect to ψ_n and ψ_m and consequently the baseline information that is already in the market.

$$VR_n = \frac{Var(v|n,m)}{Var(v|n,m,r_n^*)} = 1 + \frac{\sigma_v^2 \sigma_{\epsilon_n}^2 \psi_{r_n}^*}{\sigma_{\epsilon_n}^2 \sigma_{\eta_n}^2 + \sigma_v^2 \sigma_{\eta_n}^2 + \sigma_v^2 \sigma_{\epsilon_n}^2 (1 - \psi_{r_n}^*)} \tag{6}$$

The next Corollary describes the effects of changes in the information endowment on the managerial bias and value relevance. Note that the discussion is based on the assumption that the manager has an incentive to inflate the valuation, $\mu_\theta > 0$. For a deflating incentives, the signs of the comparative statics reverse:

Corollary 1 *If pieces of the information endowment change, managerial bias and value relevance change as follows (for $\mu_\theta > 0$):*

	(i) c	(ii) μ_θ	(iii) σ_θ	(iv) σ_v		(v) σ_{ϵ_n}		(vi) σ_{ϵ_m}	(vii) σ_{η_n}
Eb_n	$-$	$+$	$-$	(a) $+$	(b) $+-$	(a) $+$	(b) $+-$	$+$	$-$
VR_n	$+$	/	$-$	$+$		$+$		$+$	$-$

Table 1: Overview of comparative statics of the expected bias $\mu_\theta \psi_{r_n}/2c$ and value relevance (VR) $Var(v \mid n, m)/Var(v \mid n, m, r_n)$ with respect to increasing managerial costs c, increasing standard deviations of: the manager's personal emphasis on the market price σ_θ, firm value σ_v, noise of the public signals n and m (i.e., σ_{ϵ_n} and σ_{ϵ_m}), and noise of the private signal σ_{η_n}, as well as an increasing mean of the manager's personal emphasis on the market price μ_θ. The symbols $+$ and $-$ depict an increase and a decrease, respectively.

Equations (5) and (6) show that both the expected bias and value relevance are functions of ψ_{r_n}, which is an important component for the comparative statics analysis. More market attention on the report increases the manager's incentives to bias as well as the value relevance of the information. However, as we will see the results nonetheless differ.

The first series of findings is in line with Fischer and Verrecchia (2000). (i) A stricter reporting oversight c directly increases the manager's cost of biasing, which reduces the bias. However, a higher c also implies that the market increases the emphasis on the manager's report, ψ_{r_n}, which increases the manager's benefits from misreporting. This first direct effect always prevails and the expected bias decreases with higher misreporting cost. In contrast, c affects value relevance only via the positive indirect effect on ψ_{r_n}. Thus, value relevance always increases in c.

An increase in (ii) μ_θ and (iii) σ_θ influence the manager's benefits from biasing. A higher expected participation in the market price, μ_θ, increases the manager's desire for a higher report and increases the bias. Value relevance is not affected by a mean increase in θ. In contrast, more opacity about the manager's incentives, σ_θ, decreases the bias. The reason is that investors are unsure what the bias would be and put a lower focus on the report, which reduces the manager's incentive to deviate from the required standard. This same effect also reduces value relevance.

The second series of results contains new implication due to the interplay of public and private information. (iv) An increase in the noise of the fundamental value, σ_v, has a twofold effect on the bias via ψ_{r_n}. First, the higher uncertainty directly increases the market's need for information, which is why it pays more attention on the manager's report. Second, also the manager reacts to a higher σ_v, who increases the weight on private and public information and decreases the weight on her own bias coefficient to better reflect what the standard requires. The market's reaction to the manager's adjustment depend on the strength of the corporate governance system. We define the corporate governance system as the marginal cost of biasing relative to the transparency of the managerial incentives, $g = c/\sigma_\theta$. Higher marginal biasing cost, which indicate a stronger reporting oversight (higher c), and higher transparency about the managerial incentives (lower σ_θ) improve the corporate governance system (g increases). In case (a), the corporate governance system is weak with $g < \overline{g}_{\sigma_v}$, and the information noise in the report is generally high. Then, the market's demand for precise information is not satisfied, yet. Therefore, it further increases the attention on the manager's report to utilize the manager's private information. In case (b), the corporate governance system is strong with $g > \overline{g}_{\sigma_v}$ and the information noise in the report is low. Then, the market perceives the manager's increase of private and public information as overcorrection and de-emphasizes the attention to the manager's report. For lower values of σ_v, both the direct and indirect effects are positive and the bias always increases. For higher values of σ_v the indirect effect turns negative. When $g < \overline{g}_{\sigma_v}$, the negative effect is stronger than the direct effect, resulting in a hump-shaped form, where the bias first increases in σ_v and then decreases. When it comes to value relevance, these complex effects with respect to ψ_{r_n} also occur but are suppressed by the direct effect of σ_v on VR. If σ_v increases, the report can contribute more to the price efficiency of the report and value relevance increases. In the empirical sense, higher uncertainty of the fundamental value could exist when the underlying assets are intangibles.

(v) Similar forces occur for an increase in the noise of the public information for the required approach by the standard, σ_{ϵ_n}. The market increases the weight on the manager's report in response to the higher uncertainty. The manager increases her own weight on private information, which the market either welcomes and increases the weight further in case (a) for a weak corporate governance system, $g < \overline{g}_{\sigma_{\epsilon_n}}$, or perceives as overcorrection and decreases the weight in case (b) for a strong corporate governance system, $g > \overline{g}_{\sigma_{\epsilon_n}}$. Value relevance increases in the noise because the report becomes a more important information source.

(vi) While a similar logic applies with respect to the public noise regarding the disregarded measurement approach, σ_{ϵ_m}, there is one important difference. In equilibrium, the manager does not use public or private information weights in the reporting, since it is also not required by the standard. Thus, there is no indirect effect via the manager's adjustment of the information content. As a result, ψ_{r_n} always increases in σ_{ϵ_m}, which increases both the expected bias and value relevance.

Lastly, (vii) higher noise in the private information σ_{η_n} reduces the bias. The manager decreases the weight on private information and investors learn less from the report. Thus, biasing becomes less attractive and the value relevance of the report decreases.[8]

The results demonstrate that changes in public and private information as well as the underlying fundamentals of the evaluated item affect the qualitative characteristics of accounting standards differently. More precise public information does not always imply that the item can be represented more faithfully since the bias can increase as well as decrease. Value relevance, instead, shows the expected results since more noise in the public information increases the usefulness of the report for investors and more noise in private information decreases the usefulness.

4 Comparing current and alternative standard

The previous section showed the influence of changes in the information environment to measuring and reporting fair value. However, for a given economic situation, it is unlikely that the various measurement approaches have the exact same properties and lead to identical valuations. The current standards on fair value in US-GAAP

[8]Similar to results (i)-(iii), part (vii) is also in line with Fischer and Verrecchia (2000).

(ASC 820) and IFRS (IFRS 13) provide clear instructions and require the users to apply the valuation technique which maximizes the observable inputs and minimizes the unobservable inputs. Depending on the asset, this could either be the market approach, x, where the conditional expected value equals

$$E[v|x, s_x] = \beta_{x,0} + \beta_{s_x} \left(s_x + \frac{\beta_x}{\beta_{s_x}} x \right), \tag{7}$$

or the income approach, y, where the conditional expected value equals

$$E[v|y, s_y] = \beta_{y,0} + \beta_{s_y} \left(s_y + \frac{\beta_y}{\beta_{s_y}} y \right). \tag{8}$$

In terms of our model, this means that the manager must apply the market approach by using x and s_x if the weight on the public information available, β_x, in relation to the weight on the private information available, β_{s_x}, is greater than the same relation following the income approach by using y and s_y:

$$\left| \frac{\beta_x}{\beta s_x} \right| \geq \left| \frac{\beta_y}{\beta s_y} \right|. \tag{9}$$

The following proposition expresses the fair value standard's requirements in terms of fundamental parameters:

Proposition 2 *The fair value standards from IFRS and US-GAAP require the preparer to use measurement approach x and not y if it is true that*

$$\frac{\sigma^2_{\epsilon_x}}{\sigma^2_{\eta_x}} \leq \frac{\sigma^2_{\epsilon_y}}{\sigma^2_{\eta_y}}.$$

Proof in the Appendix.

The proposition uses the ratio of the noise terms from the public information over the noise terms from the manager's private information. If this ratio is smaller for measurement approach x than y, measurement approach x delivers more observable inputs. When classifying assets into categories, the term "Level 2-asset" is generally used for an asset as long as it is based on observable inputs, independent of the size of the adjustment necessary. In this sense, a "Level 3-asset" only contains unobservable inputs. The above proposition does not distinguish between these cases. Measurement approach y could lead to a Level 3-asset if the noise of the public information, y, would simply approach infinity, $\sigma^2_{\epsilon_y} \to \infty$. Thus, our results hold for both comparing inputs which lead to either two Level 2-assets or one Level 2 and one Level 3-asset.

In the next step, we evaluate the measurement approaches in terms of the qualitative characteristics: faithful representation (expected bias) and value relevance. Following the properties of Proposition 2, we assume an environment where the standard requires the use of the market approach x, which we label the "current standard". Income approach y is the "alternative standard". We identify four different areas:

I) Classic Trade-off: $Eb_x < Eb_y$ and $VR_x < VR_y$

II) Classic Trade-off reversed: $Eb_x > Eb_y$ and $VR_x > VR_y$

III) Y dominates: $Eb_x > Eb_y$ and $VR_x < VR_y$

IV) X dominates: $Eb_x < Eb_y$ and $VR_x > VR_y$.

I) Classic Trade-off describes the outcome that could be expected based on the properties of the inputs. A focus on observable over unobservable inputs provides a more reliable but potentially less relevant measure (Laux and Leuz 2009). In contrast, *II) Classic Trade-off reversed* describes the opposite scenario where more observable inputs lead to a more relevant but a less reliable measure. In the remaining two areas, one standard dominates in terms of both characteristics, more relevance and more reliability, whereby standard Y dominates in *III)* and standard X in *IV)*. Figure 1 shows a comparison of the current and the alternative standards indicating the four potential areas depending on a change in σ_{ϵ_x} and σ_{η_x}.

Two overall observations are striking. The first observation is that the classic trade-off (region I)) makes up for only a fraction of cases, when public signal noise is rather low and private signal noise is high. Depending on the parameters, other output constellations are possible. In particular, for high noise in the public signal and low noise in the private signal, the classic trade-off reversed emerges (region II)). For medium levels of public and private noise as well as extremely low levels of public noise, the alternative standard dominates in both qualitative characteristics (region III)). The reason lies in the asymmetry of both measurement approaches, most apparent from Panel B. The bias and value relevance both decrease in private noise for measurement approach x, whereas approach y is unaffected. Similarly, in Panel A, comparative statics from Corollary 1 part (v) apply to approach x and from part (vi) to approach y.

The second observation is that region IV), where the current standard dominates, does not exists.

In addition to the above mentioned results, Figure 2 shows that the areas are robust to variation and the results are generalizable.

Proposition 3 *If* $\sigma^2_{\eta_x} < \sigma^2_{\eta_y} \wedge \frac{\sigma^2_{\epsilon_x}}{\sigma^2_{\eta_x}} \leq \frac{\sigma^2_{\epsilon_y}}{\sigma^2_{\eta_y}} \wedge \sigma^2_v < \frac{\sigma^2_{\epsilon_x}\sigma^2_{\epsilon_y}\left(\sigma^2_{\eta_y}-\sigma^2_{\eta_x}\right)}{\sigma^2_{\epsilon_y}\sigma^2_{\eta_x}-\sigma^2_{\epsilon_x}\sigma^2_{\eta_y}}$ *then we always have area II) Classic Trade-off reversed.*

If $\sigma^2_{\eta_x} < \sigma^2_{\eta_y} \wedge \frac{\sigma^2_{\epsilon_x}}{\sigma^2_{\eta_x}} \leq \frac{\sigma^2_{\epsilon_y}}{\sigma^2_{\eta_y}} \wedge \sigma^2_v > \frac{\sigma^2_{\epsilon_x}\sigma^2_{\epsilon_y}\left(\sigma^2_{\eta_y}-\sigma^2_{\eta_x}\right)}{\sigma^2_{\epsilon_y}\sigma^2_{\eta_x}-\sigma^2_{\epsilon_x}\sigma^2_{\eta_y}}$ *then we have*

area I) Classic Trade-off for $g < \bar{\bar{g}}$

area III) Y dominates for $\bar{\bar{g}} < g < \bar{\bar{\bar{g}}}$

area II) Classic Trade-off reversed $g > \bar{\bar{\bar{g}}}$ *.*

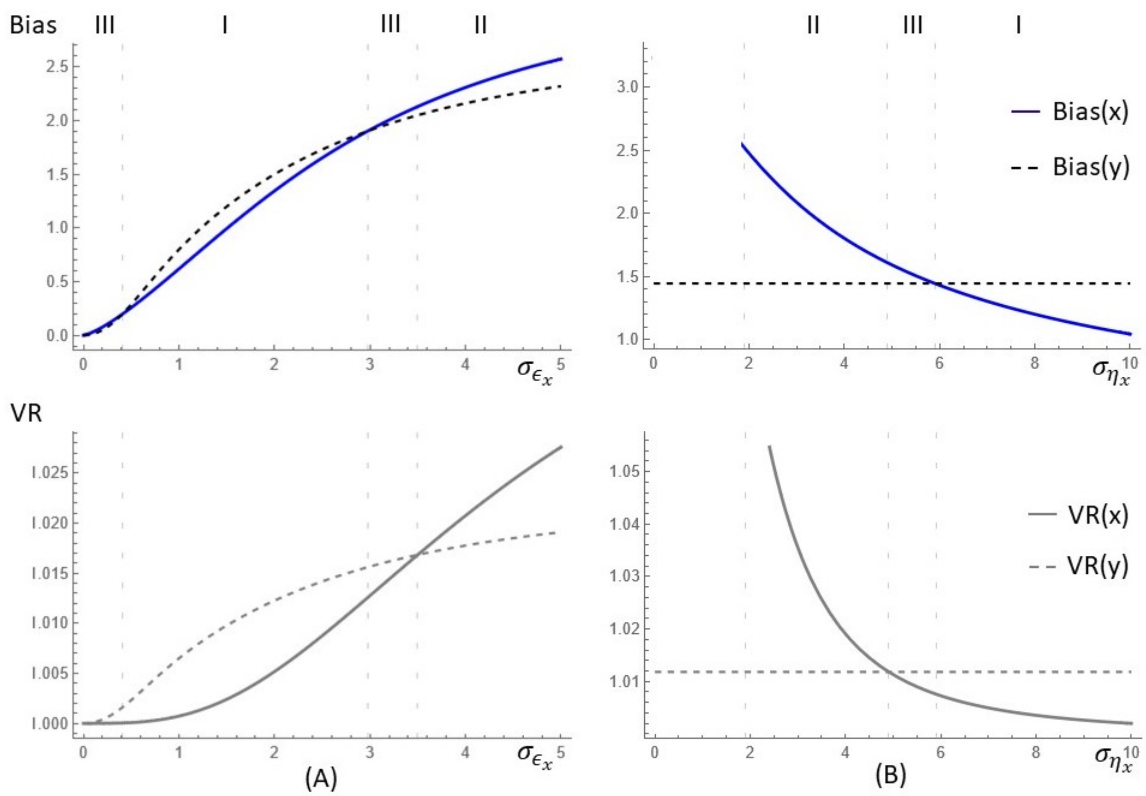

Figure 1: Expected manager's bias and value relevance (VR) for the Level-2 and the Level-3 Standard depending on the uncertainty about (A) the public information, σ_{ϵ_x} and (B) the private information, σ_{η_x}.
The parameter values are $\mu_v = 4$, $\mu_\theta = 3$, $\sigma_\theta = 3$, $\sigma_{\eta_y} = 10$, $\sigma_{\epsilon_y} = 10$, $\sigma_v = 4$, $c = 0.1$, and $\sigma_{\eta_x} = 7.25$ (in panel (A)), and $\sigma_{\epsilon_x} = 1.9$ (in panel (B)).

If $\sigma^2_{\eta_x} > \sigma^2_{\eta_y} \wedge \frac{\sigma^2_{\epsilon_x}}{\sigma^2_{\eta_x}} \leq \frac{\sigma^2_{\epsilon_y}}{\sigma^2_{\eta_y}}$ *then if* $\psi^*_{r_x} < \psi^*_{r_y}$ *then* $VR_x < VR_y$*, hence, there exists to region IV.*

For low uncertainty about the fundamental value of the asset and/or strong corporate governance, the current standard delivers a higher value relevance but is potentially more biased than the alternative, resulting in the classic trade-off reversed (area II). Only for sufficiently high uncertainty about the fundamental value and weak corporate governance systems, the classic trade-off prevails (area I). Instead, when the corporate governance system increases to moderate, the alternative standard dominates (area III) in both qualitative characteristics.

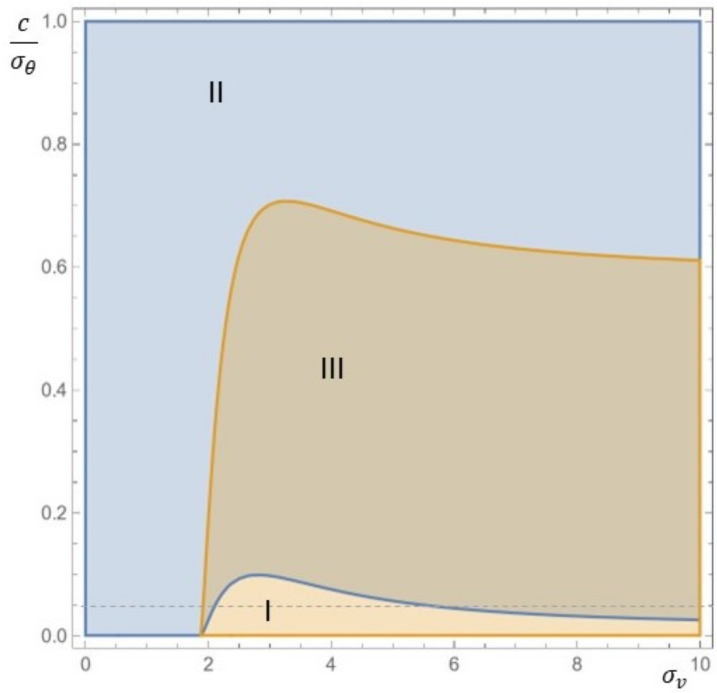

Figure 2: Region Plot
The parameter values are $\mu_{\epsilon_x} = 2$, $\mu_{\epsilon_y} = 2$, $\mu_v = 4$, $\mu_\theta = 3$, $\sigma_\theta = 3$, $\sigma_{\eta_y} = 10$, $\sigma_{\epsilon_y} = 10$, $\sigma_v = 4$, $c = 0.1$, $\sigma_{\eta_x} = 7.25$, and $\sigma_{\epsilon_x} = 1.9$.

5 Extensions

5.1 Using all available information

Viewing the comparison of current and alternative standard raises the question whether a mix of both would render a better result. Therefore, we apply in this section the theoretical concept when all information from the market approach and income approach is used to generate one fair value measure. Hence, the standard would require the conditional expected value

$$E[v|x, s_x, y, s_y] = \beta_{xy,0} + \beta_{s_x}\left(s_x + \frac{\beta_x}{\beta_{s_x}}x\right) + \beta_{s_y}\left(s_y + \frac{\beta_y}{\beta_{s_y}}y\right). \quad (10)$$

Accordingly, the manager's reporting strategy and the market's pricing strategy can be expressed as

$$r^*_{xy} = \arg\max_{r_{xy}} E[\theta \cdot \widehat{P}_{xy} - c(r_{xy} - E[v|x, s_x, y, s_y])^2|x, y, s_x, s_y, \theta] \quad (11)$$

$$P_{xy} = E\left[v|x, y, r_{xy}\right] \quad (12)$$

We establish the unique linear equilibrium in the same fashion as before.[9] One could think that including all information available leads to a higher value relevance in all cases. However, Figure 3 shows that this is not true. For example, when the manager's private information of the market approach x is very noisy, value relevance is higher with approach y. The reason is related to Corollary 1 part (vii). Considering approach x and the full information approach xy, more noise in the private signal, σ_{η_x}, lowers the market's attention to the manager's report and consequently lowers the expected bias and value relevance. However, σ_{η_x} does not affect approach y. Thus, with sufficiently high noise, value relevance of x and xy falls below y. Therefore, price efficiency benefits from excluding noisy private signals.

5.2 Specialist Auditor

As measuring fair value is a complex process, it often requires the involvement of valuation specialists. The specialists could either be employed directly by the manager

[9] Proofs upon request.

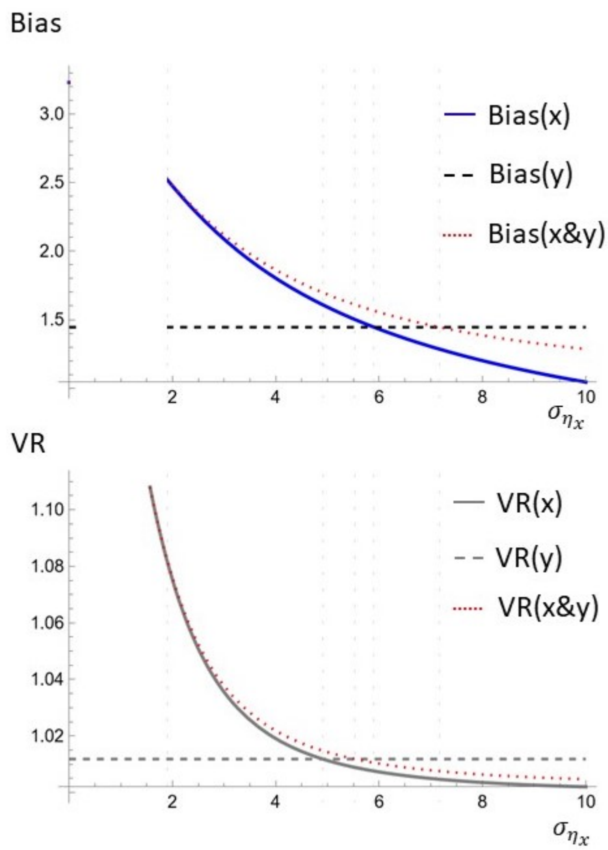

Figure 3: Expected manager's bias and value relevance (VR) for the Level-2 and the Level-3 Standard in setting where all available information can be used depending on the uncertainty about the private information, σ_{η_x}.
The parameter values are $\mu_{\epsilon_x} = 2$, $\mu_{\epsilon_y} = 2$, $\mu_v = 4$, $\mu_\theta = 3$, $\sigma_\theta = 3$, $\sigma_{\eta_y} = 10$, $\sigma_{\epsilon_y} = 10$, $\sigma_v = 4$, $c = 0.1$, and $\sigma_{\epsilon_x} = 1.9$.

as an input, when the manager has insufficient valuation expertise or they can be employed by the financial statement auditor to verify the reported fair value. In this section, we address the latter. For this purpose, we extend the baseline model and include the possibility of verifying the manager's report with the involvement of a specialist on the side of the auditor. First, the auditor determines the audit effort level for the audit process, which can be thought of as a budget for the specialist, and the manager receives the private signals (as before).[10] Then, the

[10]In a similar fashion, Caskey et al. (2010) determine the audit committees' ex-ante effort level.

auditor receives the unaudited report from the manager, r_M, and generates with the specialist the respective signal in the audit process about the manager's private information depending on the standard's requirement, $s_{Ax} = s_x + \eta_{Ax}$ or $s_{Ay} = s_y + \eta_{Ay}$ with uncorrelated noise terms $\eta_{Ax} \sim N(0, \sigma^2_{\eta_{Ax}})$ or $\eta_{Ay} \sim N(0, \sigma^2_{\eta_{Ay}})$. A higher effort or specialist budget reduces the noise of these private signals. Based on this information, the auditor reports r_A, which is the final report transmitted to the market. Thus, the market pricing no longer relies on r_M, which is now unobservable to the market, but on r_A:

$$P_{r_A} = E\left[v|x, y, r_A\right] \tag{13}$$

The manager's rationale remains unchanged with the exception that the market price is based on r_A:[11]

$$r^*_M = \arg\max_{r_M} U_M = \arg\max_{r_M} E[\theta \cdot \widehat{P}(x, y, r_A) - c(r_M - E[v|n, s_n])^2|x, y, s_n, \theta] \tag{14}$$

The auditor receives fee, F, for the audit, which is exogenous and large enough to cover the auditor's costs. The auditor faces liability consequences, L, whenever the audited report deviates from the standard's requirement. L captures the liability payment as well as the (exogenous) probability of being held liable for an audit mistake. The liability is more likely and greater in amount if the deviation becomes larger. Further, the auditor has direct cost from providing the budget to the specialist and increasing the precision (or decreasing the variance) of the audit signal, $K(\sigma_{\eta_{An}})$.

[11] Other cost functions for the managerial bias might be feasible as well, i.e., $c(r_M - \widehat{r}_A)^2$ or $c(\widehat{r}_A - E[v|x, s_{Mx})^2$. We decided to use the same cost function in the extension and baseline setting to better compare the results.

For simplification, we assume that the auditor only generates the signal about the manager's private information that is connected to the measurement approach required by the standard.[12] The budget is a convex function in audit signal precision with $dK(\sigma_{\eta_{An}})/d\sigma_{\eta_{An}} < 0$ and $d^2K(\sigma_{\eta_{An}})/d\sigma^2_{\eta_{An}} < 0$ and $K(\sigma_{\eta_{An}} \to \infty) = 0$. Thus, the auditor's utility is given by

$$U_A = F - (r_A - E[v|n, s_n])^2 L - K(\sigma_{\eta_{An}}), \tag{15}$$

The auditor provides her own fair value estimate based on her private signals which the auditor reports accordingly. The engagement partner usually does not conduct the fair value estimate herself, but relies on valuation specialists, either from in-house or third party consultants. Nevertheless, the liability remains with the engagement partner (PCAOB AS No.10).[13] The auditor chooses the optimal report, r_A, that maximizes the utility in line with (15), which consists of a fixed fee, expected liability and budget cost:

$$r_A^* = \arg\max_{r_A} U_A = \arg\max_{r_A} F - E[(r_A - E[v|n, s_n])^2 L - K(\sigma_{\xi_{Ax}})|x, y, s_{An}, r_M] \tag{16}$$

We establish the unique linear equilibrium in the same fashion as before.[14]

[12]Nummerical solutions indicate that the weight on the private information not required by the standard, s_{Am}, is neglectable.

[13]The valuation specialists suggest a range of acceptable values, which can be adjusted by the engagement partner depending on other findings in the audit. If the manager's fair value falls outside of this range, the auditor should treat the difference as misstatements (PCAOB AS No.14). Then, the auditor would adjust the manager's report from r_M to r_A.

[14]Proofs upon request.

In the auditor setting, the reaction to the manager's expected bias

$$Eb_A = \frac{\mu_\theta \cdot \alpha_{r_M} \cdot \psi_{r_A}}{2c} \tag{17}$$

proceeds in two stages. First, the auditor receives the manager's report and filters out the bias as good as possible, α_{r_M}. Then, the market receives the filtered version from the auditor, ψ_{r_A}. Thereby, the auditor acts in the market's best interest since the alignment of report and standard is not just in the market's interests, but also in the auditor's interest to avoid liability. Value relevance is then:

$$VR_A = \frac{Var(v|x,y)}{Var(v|x,y,r_A^*)} \tag{18}$$

Changes of the information endowment influence the managerial bias and value relevance in a similar fashion as in the baseline model (compare Corollary 1). Thus, we do not repeat the insights. The interesting aspect we want to emphasize in this section is how the auditor's liability, as the primary incentive to provide a high quality audit, impacts the evaluation with respect to the qualitative characteristics of the current and the alternative fair value standard. Given liability damage payment, L, the auditor decides on the planned audit effort level based on (15) and maximizes the utility or equivalently, minimizes the expected liability and the direct effort cost by exerting effort and choosing the precision level of her private signal, $\sigma_{\eta_{A_n}}$.

An increase in liability directly increases also the auditor's precision level choice (decreases $\sigma_{\eta_{A_n}}$). Thus, higher liability reduces the probability that a bias is undetected and reduces the biasing incentives for the manager. Value relevance increases since the audited report becomes more important. Thus, we can state the following proposition:

Proposition 4 *The manager's bias decreases and value relevance increases with higher auditor liability payments (L increases)*

The results can also be seen in Figure 4. In addition, we can observe the comparison between current and alternative standard. Interestingly, with the involvement of a specialist on the auditor's side, the expected trade-off (area I)), where the current standard is reliable but not as relevant, only arises for lenient liability payments. For moderate liability regimes, the alternative standard dominates (area III). For a strict liability regimes, the current standard shows a higher value relevance but also a higher managerial bias, the reversal of what would be expected when considering more observable inputs (area II).

Thus, the interplay of market, manager, and specialist/auditor can result in varying conditions for investors when interpreting fair value measurements in varying institutional settings. Our model predicts that in low to moderate liability regimes, such as Germany or other continental European countries, value relevance of Level 3-assets are higher than Level 2-assets. In contrast, in strict liability regimes, such as the US, Level 2-assets are more value relevant. Simultaneously, the expected bias is larger for Level 2-assets in most jurisdiction, except when the liability is extremely lenient.

6 Conclusion

Current fair value standards in US-GAAP and IFRS require the preparer to use the measurement approach, which maximizes observable inputs and minimizes unobservable inputs. The standard setters' intention is to provide a reliable market-based measurement, rather than an entity-specific, measurement. This concept is supposed to constrain

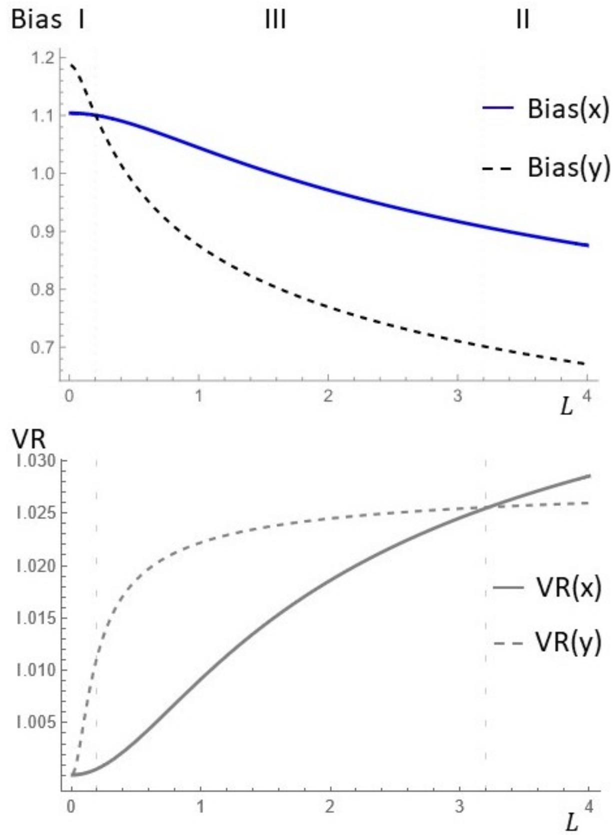

Figure 4: Expected manager's bias and value relevance (VR) for the Level-2 and the Level-3 Standard depending on the auditor's liability, L.
The parameter values are $\mu_{\epsilon_x} = 2$, $\mu_{\epsilon_y} = 2$, $\mu_v = 4$, $\mu_\theta = 3$, $\sigma_\theta = 3$, $\sigma_{\eta_y} = 10$, $\sigma_{\epsilon_y} = 10$, $\sigma_v = 4$, $c = 0.1$, $\sigma_{\eta_x} = 7.25$, and $\sigma_{\epsilon_x} = 1.9$.

the manager's discretion to bias financial reports but is possibly less relevant to investors as it conveys less private information of the manager. We apply a rational expectations model in line with Fischer and Verrecchia (2000) and introduce, in addition to the manager and the investors, an independent auditor. Based on the market price formed by the interactions of the players, we find that the trade-off for standard setters between the manager's expected bias and value relevance is not as clear-cut as commonly suggested. An alternative standard, which requires the maximization of *un*observable inputs, sometimes induces a smaller managerial bias than the current standard. In addition, this alternative can still provide a higher relevance since it carries more of the manager's private information.

From a standard setters view, one might still argue that a rigid hierarchy as in the current standard has other benefits not captured in our model, e.g. to enhance comparability of financial statements. In this case, our results at least emphasize the cost of restricting the measurement approach to maximize observable inputs under certain conditions. Comparability, however, seems to become less important to standard setters, which is reflected in the downgrading of comparability from a qualitative to an enhancing characteristic in the conceptual framework (FASB, 2018; IASB, 2018), leaving relevance and faithful representation as the only qualitative characteristics.

We further provide insights how other types of regulation, i.e., the liability environment of an auditor, impacts the standard setters trade-offs. We show that in a strict liability regimes, such as the US, the managerial bias of the current standard is higher than the bias with the alternative standard. Practitioners and scholars frequently question the auditor's contribution in the verification of fair values since it is a very complex task with only limited guidance for auditors. However, we find in our model that even an auditor with a very noisy private signal always contributes to the quality of the information in the market. Whether this benefit exceeds the costs of auditing is essentially an empirical question.

7 Appendix

Proof of Proposition 1.

The players' strategies.

a) The Manager's Linear Strategy. The manager believes that the price is a linear function of the public information: x, y and r_n:

$$\hat{P}_n(r_n, x, y) = \hat{\psi}_{0,n} + \hat{\psi}_{r_n} r_n + \hat{\psi}_n n + \hat{\psi}_m m$$

The first order-condition of maximizing manager's utility in (1) with respect to r_n leads to:

$$r_n = \frac{\theta \hat{\psi}_{r_n}}{2c} + E[v|n, s_n]$$

Since the manager's linear strategy is

$$r_n = \omega_{0,n} + \omega_n n + \omega_m m + \omega_{s_n} s_n + \omega_{s_m} s_m + \omega_{\theta,n}\theta, \tag{19}$$

the corresponding weights are $\omega_\theta = \frac{\hat{\psi}_{r_n}}{2c}$, $\omega_n = \beta_n$, $\omega_{0,n} = \beta_{0,n}$, $\omega_{s_n} = \beta_{s_n}$, $\omega_m = 0$, and $\omega_{s_m} = 0$.

b) The Market's Linear Strategy.

The market evaluates the firm with $E[v|n, m, \hat{r}_n]$ where $\hat{r}_n$ are the beliefs about manager's linear strategy given by equation (19). For calculating the conditional expected value we need the conditional density function:

$$f(v|n, m, \hat{r}_n) = \frac{f(v, n, m, \hat{r}_n)}{f(n, m, \hat{r}_n)} = \frac{f(\hat{r}_n|n, m, v) \cdot f(n|v) \cdot f(m|v) \cdot f(v)}{f(n, m, \hat{r}_n)}$$

1) $f(v)$ is given by our assumptions.

2) $f(n|v)$. We can write $f(n|v) \cdot f(m|v) = f(n, m|v)$ because once we know v, there is no additional information of x about y and vice versa and hence no correlation between x and y. Consequently, $f(n|v)$ is given by $N \sim (v + \mu_{\epsilon_n}, \sigma^2_{\epsilon_n})$ and $f(m|v)$ is given by $N \sim (v + \mu_{\epsilon_m}, \sigma^2_{\epsilon_m})$.

3) $f(\hat{r}_n|n, m, v)$.

This conditional density function is a normal distribution function with mean:

$$E[\hat{r}_n|n, m, v] = \hat{\omega}_{0,n} + \hat{\omega}_{\theta,n}\mu_\theta + \hat{\omega}_n n + \hat{\omega}_m m + \hat{\omega}_{s_n}(n - v) + \hat{\omega}_{s_m}(m - v)$$

where $E[s_n|n, m, v] = E[\epsilon_n|n, m, v] = n - v$ and $E[s_m|n, m, v] = E[\epsilon_n|n, m, v] = m - v$ follows from the equations $n = v + \epsilon_n$ and $m = v + \epsilon_m$ and the fact that η_n and η_m are independent of n, m and v and have mean of zero.

The conditional variance of the above density function is given by:

$$Var[\hat{r}_n|n, m, v] = \hat{\omega}^2_{\theta,n}\sigma^2_\theta + \hat{\omega}^2_{s_n}\sigma^2_{\eta_n} + \hat{\omega}^2_{s_m}\sigma^2_{\eta_m}$$

this follows from the fact that given n, m and v the unknown variables are η_n and η_m and θ, which are independent from each other. Remember that we can substitute s_n and s_m with $s_n = \epsilon_n + \eta_n = n - v + \eta_m$ and $s_m = \epsilon_m + \eta_m = m - v + \eta_m$ in equation (19).

4) $f(n, m, \hat{r}_n)$.

The joint distribution in the denominator is the integral of the full numerator over v. Thus, we have all ingredients and can determine $E[v|n, m, \hat{r}_n]$:

$$E[v|n, m, \hat{r}_n] = \int v \cdot f(v|n, m, \hat{r}_n)dv.$$

Based on this expected value, we can calculate the weights of the markets on the respective information components, $\psi_{0,n}, \psi_n,\psi_m$, and ψ_{r_n}.

Uniqueness of the equilibrium.

The equilibrium is defined by an equation system with 10 equation and 10 unknowns:

$$\omega_{\theta,n} = \frac{\hat{\psi}_{r_n}}{2c} \tag{20}$$

$$\omega_n = \frac{COV_{nv}}{\sigma_n^2} \cdot (\frac{1}{1-\rho_{ns_n}}) \tag{21}$$

$$\omega_{s_n} = -\frac{COV_{nv}}{\sigma_n^2} \cdot (\frac{1}{1-\rho_{ns_n}}) \frac{Cov_{ns_n}}{\sigma_{s_n}^2} \tag{22}$$

$$\omega_m = 0 \tag{23}$$

$$\omega_{s_m} = 0 \tag{24}$$

$$\omega_0 = \mu_v - \omega_n \mu_n + \omega_{s_n} \mu_{s_n} \tag{25}$$

$$\psi_{r_n} = \frac{\sigma_v}{\sigma_{r_n}} \frac{\rho_{vr_n} + \rho_{nv}\rho_{mr_n}\rho_{mn} + \rho_{nm}\rho_{nr_n}\rho_{mv} - \rho_{mr_n}\rho_{mv} - \rho_{vr_n}\rho_{mn}^2 - \rho_{nr_m}\rho_{nv}}{1-\rho_{mr_n}^2 - \rho_{nr_n}^2 - \rho_{nm}^2 + 2\rho_{mr_n}\rho_{nr_n}\rho_{nm}} \tag{26}$$

$$\psi_n = \frac{\sigma_v}{\sigma_n} \frac{\rho_{nv} + \rho_{nr_n}\rho_{mr_n}\rho_{mv} + \rho_{vr_n}\rho_{mr_n}\rho_{mx} - \rho_{mn}\rho_{mv} - \rho_{nv}\rho_{mr_n}^2 - \rho_{vr_n}\rho_{nr_n}}{1-\rho_{mr_n}^2 - \rho_{nr_n}^2 - \rho_{nm}^2 + 2\rho_{mr_n}\rho_{nr_n}\rho_{nm}} \tag{27}$$

$$\psi_m = \frac{\sigma_v}{\sigma_m} \frac{\rho_{mv} + \rho_{mr_n}\rho_{nr_n}\rho_{nv} + \rho_{vr_n}\rho_{nr_n}\rho_{mn} - \rho_{nm}\rho_{nv} - \rho_{mv}\rho_{nr_n}^2 - \rho_{vr_n}\rho_{mr_n}}{1-\rho_{mr_n}^2 - \rho_{nr_n}^2 - \rho_{nm}^2 + 2\rho_{mr_n}\rho_{nr_n}\rho_{nm}} \tag{28}$$

$$\psi_0 = \mu_v - \psi_m \mu_m - \psi_n \mu_n - \psi_{r_n} \mu_{r_n} \tag{29}$$

$\rho_{ij} = \frac{COV_{ij}}{\sigma_i \sigma_j}$ is the correlation coefficient between i and j with $i, j \in \{x, y, v, r_n\}$. Because of the information structure the covariances are given by: $COV_{nm} = COV_{nv} = COV_{mv} = \sigma_v^2$, $COV_{nr_n} = \omega_n \sigma_n^2 + \omega_m \sigma_v^2 + \omega_{s_n} \sigma_{\epsilon_n}^2$, $COV_{mr_n} = \omega_m \sigma_m^2 + \omega_n \sigma_v^2 + \omega_{s_m} \sigma_{\epsilon_m}^2$ and $COV_{vr_n} = \omega_n \sigma_v^2 + \omega_m \sigma_v^2$. The variances are given by: $\sigma_n^2 = \sigma_v^2 + \sigma_{\epsilon_n}^2$,$\sigma_m^2 = \sigma_v^2 + \sigma_{\epsilon_m}^2$

and $\sigma_{r_n}^2 = \omega_n^2\sigma_n^2 + \omega_m^2\sigma_m^2 + \omega_{s_n}^2(\sigma_{\epsilon_n}^2 + \sigma_{\eta_n}^2) + \omega_{s_m}^2(\sigma_{\epsilon_m}^2 + \sigma_{\eta_m}^2) + \omega_{\theta,n}^2\sigma_\theta^2 + 2\omega_n\omega_m\sigma_v^2 + 2\omega_n\omega_{s_n}\sigma_{\epsilon_n}^2 + 2\omega_m\omega_{s_m}\sigma_{\epsilon_m}^2$. By inserting these covariance and variance in equation (26) we get:

$$\psi_{r_n} = \frac{-\sigma_v^2\sigma_{\epsilon_n}^2\sigma_{\epsilon_m}^2(\omega_{s_n} + \omega_{s_m})}{\sigma_v^2\sigma_{\epsilon_n}^2\sigma_{\epsilon_m}^2(\omega_{s_n} + \omega_{s_m})^2 + (\sigma_{\epsilon_n}^2\sigma_{\epsilon_m}^2 + \sigma_{\epsilon_n}^2\sigma_v^2 + \sigma_{\epsilon_m}^2\sigma_v^2)(\sigma_{\eta_n}^2\omega_{s_n}^2 + \sigma_{\eta_m}^2\omega_{s_m}^2 + \sigma_\theta^2\omega_{\theta,n}^2)} \tag{30}$$

If we insert equations (24), (22) and (**??**) in the above equation we receive the equation which defines the equilibrium:

$$\psi_{r_n} = \frac{A}{B + C\psi_{r_n}^2} \tag{31}$$

with $A = \frac{\sigma_v^4\sigma_{\epsilon_n}^4\sigma_{\epsilon_m}^2}{\sigma_{\epsilon_n}^2\sigma_{\eta_n}^2 + \sigma_v^2\sigma_{\epsilon_n}^2 + \sigma_v^2\sigma_{\eta_n}^2}$, $B = \frac{\sigma_v^4\sigma_{\epsilon_n}^6\sigma_{\epsilon_m}^2\sigma_{\eta_n}^2 + \sigma_v^6\sigma_{\epsilon_n}^4\sigma_{\epsilon_m}^2\sigma_{\eta_n}^2 + \sigma_v^6\sigma_{\epsilon_n}^6\sigma_{\epsilon_m}^2 + \sigma_v^6\sigma_{\epsilon_n}^6\sigma_{\eta_n}^2}{(\sigma_{\epsilon_n}^2\sigma_{\eta_n}^2 + \sigma_v^2\sigma_{\epsilon_n}^2 + \sigma_v^2\sigma_{\eta_n}^2)^2}$ and $C = \frac{\sigma_\theta^2}{4c^2}(\sigma_{\epsilon_n}^2\sigma_{\epsilon_m}^2 + \sigma_v^2\sigma_{\epsilon_n}^2 + \sigma_v^2\sigma_{\epsilon_m}^2)$

Multiply both sides with the denominator leads to:

$$F(\psi_{r_n}) \equiv C\psi_{r_n}^3 + B\psi_{r_n} - A = 0 \tag{32}$$

We define the left side of (32) as $F(\psi_{r_n})$ with $F(\psi_{r_n}^* = 0)$ in equilibrium. From equation (32) it is clear that ψ_{r_n} is larger than zero as $F(\psi_{r_n}) < 0$ if $\psi_{r_n} < 0$. Note also that $lim_{\psi_{r_n} -> 0} F(\psi_{r_n}) = -A$ and $lim_{\psi_{r_n} -> \frac{A}{B}} F(\psi_{r_n}) = C(\frac{A}{B})^3 > 0$ and $\frac{\partial F(\psi_{r_n})}{\partial \psi_{r_n}} = 3C\psi_{r_n}^2 + B > 0$. Hence, as function $F(\psi_{r_n})$ is continuous, strictly increasing, $F(0) < 0$ and $F\left(\frac{A}{B}\right) > 0$, there exist a single solution for ψ_{r_n} in the interval $\psi_{r_n} \in \left[0, \frac{A}{B}\right]$.

Proof of Corollary 1

Expected Bias

First, we show the proofs for the expected bias, which is given by equation: $\frac{\mu_\theta \cdot \psi^*_{r_n}}{2c}$. Because of the implicit function theorem, we know that the partial derivatives for $\psi^*_{r_n}$ with respect to i, $i \in \left\{c, \sigma_\theta, \sigma_v, \sigma_{\epsilon_x}, \sigma_{\epsilon_y}, \sigma_{\eta_{Mx}}\right\}$ are given by:

$$\frac{\partial \psi^*_{r_n}}{\partial i} = -\frac{\frac{\partial F(\psi_{r_n})}{\partial i}}{\frac{\partial F(\psi_{r_n})}{\partial \psi_{r_n}}}$$

Part (i): Differentiating the expected bias with respect to c we get:

$$\frac{\partial Eb}{\partial c} = \frac{\mu_\theta}{2}\left(\frac{\frac{\partial \psi^*_{r_n}}{\partial c}c - \psi^*_{r_n}}{c^2}\right) = \frac{\mu_\theta}{2c^2}\left(\frac{-C\psi^{*3}_{r_n} - B\psi^*_{r_n}}{3C\psi^{*2}_{r_n} + B}\right) < 0$$

This derivative is smaller than zero as $\psi^*_{r_n} > 0$. The second equation follows from the implicit function theorem: $\frac{\partial \psi^*_{r_n}}{\partial c} = \frac{2\psi^{*3}_{r_n}C}{c}/(3C\psi^2_{r_n} + B)$.

Part (ii): The expected bias is increasing in μ_θ:

$$\frac{\partial Eb}{\partial \mu_\theta} = \frac{\psi^*_{r_n}}{2c} > 0.$$

Part (iii): The expected bias is decreasing in σ_θ:

$$\frac{\partial Eb}{\partial \sigma_\theta} = \frac{\mu_\theta}{2c}\frac{\partial \psi^*_{r_n}}{\partial \sigma_\theta} = \frac{\mu_\theta}{2c}\frac{\frac{-2C}{\sigma_\theta}\psi^3_{r_n}}{3C\psi^2_{r_n} + B} < 0$$

Part (iv): Remember that $g = \frac{c}{\sigma_\theta}$. Then, the expected bias increases in σ_v independent of the level of σ_v if

$$g < \overline{g}_{\sigma_v} \equiv \frac{\sigma^2_{\epsilon_m}\left(\sigma^2_{\epsilon_n} + \sigma^2_{\eta_n}\right)^2\left(2\sigma^2_{\epsilon_m}\sigma^2_{\eta_n} + \sigma^2_{\epsilon_n}\sigma^2_{\epsilon_m} + \sigma^2_{\epsilon_n}\sigma^2_{\eta_n}\right)^{\frac{3}{2}}}{2\sigma^3_{\epsilon_n}\sigma^2_{\eta_n}\left(\sigma^4_{\epsilon_n}\sigma^4_{\epsilon_m} + \sigma^2_{\epsilon_n}\sigma^2_{\epsilon_m}\left(2\sigma^2_{\epsilon_n} + 3\sigma^2_{\epsilon_m}\right)\sigma^2_{\eta_n} + 2\sigma^4_{\eta_n}\left(\sigma^2_{\epsilon_n} + \sigma^2_{\epsilon_m}\right)^2\right)}$$

If $g > \overline{g}_{\sigma_v}$ then the expected bias increases (decreasing) in σ_v if $\sigma_v < (>)\underline{\sigma}_v$ where $\underline{\sigma}_v$

is defined by $g = G(\underline{\sigma}_v, \sigma_{\epsilon_n}, \sigma_{\eta_n}, \sigma_{\epsilon_m})$.

The above condition can also be written in the following way: for a given σ_v expected bias increases (decreases) in σ_v if

$$g < (>) G(\sigma_v, \sigma_{\epsilon_n}, \sigma_{\eta_n}, \sigma_{\epsilon_m}). \tag{33}$$

Equation (32) can also be solved explicitly by using Cardano's Formula. By doing so, we can solve expected bias in equilibrium explicitly. Inequality (33) follows from differentiating this expected bias with respect to σ_v. G has the following characteristics: decreasing in σ_v, $lim_{\sigma_v -> 0} G = \infty$ and $lim_{\sigma_v -> \infty} G = \overline{g}_{\sigma_v}$.

Part (v): If inequality (33) holds, then the expected bias increases (decreases) in σ_{ϵ_n}. Note that G has the following characteristics: decreasing in σ_{ϵ_n} and $lim_{\sigma_{\epsilon_n} -> \infty} G = \overline{g}_{\sigma_{e_n}}$. Hence, the expected bias increases in σ_{ϵ_n} independent of the level of σ_{ϵ_n} if

$$g < \overline{g}_{\sigma_{e_n}} \equiv \frac{\sigma_{\epsilon_m}^2 \left(\sigma_v^2 + \sigma_{\eta_n}^2\right)^2 \left(2\sigma_{\epsilon_m}^2 \sigma_{\eta_n}^2 + \sigma_v^2 \sigma_{\epsilon_m}^2 + \sigma_v^2 \sigma_{\eta_n}^2\right)^{\frac{3}{2}}}{2\sigma_v^3 \sigma_{\eta_n}^2 \left(\sigma_v^4 \sigma_{\epsilon_m}^4 + \sigma_v^2 \sigma_{\epsilon_m}^2 \left(2\sigma_v^2 + 3\sigma_{\epsilon_m}^2\right) \sigma_{\eta_n}^2 + 2\sigma_{\eta_n}^4 \left(\sigma_v^2 + \sigma_{\epsilon_m}^2\right)^2\right)}.$$

If $g > \overline{g}_{\sigma_{e_n}}$ then the expected bias increases in σ_{ϵ_n} if $\sigma_{\epsilon_n} < (>) \underline{\sigma}_{\epsilon_n}$ where $\underline{\sigma}_{\epsilon_n}$ is defined by $g = G(\sigma_v, \underline{\sigma}_{\epsilon_n}, \sigma_{\eta_n}, \sigma_{\epsilon_m})$.

Part (vi): The expected bias increases in σ_{ϵ_m}:

$$\frac{\partial Eb}{\partial \sigma_{\epsilon_m}} = \frac{\mu_\theta}{2c} \frac{\partial \psi_{r_n}^*}{\partial \sigma_{\epsilon_m}} = -\frac{\mu_\theta}{2c} \frac{\frac{2\sigma_v^4 \sigma_{\epsilon_n}^4 \sigma_{\epsilon m}(-1+\psi_{rn})}{\sigma_{\epsilon_n}^2 \sigma_{\eta_n}^2 + \sigma_v^2 \sigma_{\epsilon_n}^2 + \sigma_v^2 \sigma_{\eta_n}^2} + \frac{(\sigma_v^2 + \sigma_{\epsilon_n}^2)\epsilon_m \sigma_\theta^2 \psi_{r_n}^3}{2c^2}}{3C\psi_{r_n}^2 + B}$$

$$= -\frac{\mu_\theta}{2c} \frac{\frac{2\sigma_{\epsilon m}(-\sigma_v^6 \sigma_{\epsilon_n}^8 \sigma_{\eta_n}^2 - \sigma_v^8 \sigma_{\epsilon_n}^6 \sigma_{\eta_n}^2 - \sigma_v^8 \sigma_{\epsilon_n}^8 (1-\psi_{rn})))}{(\sigma_{\epsilon_n}^2 \sigma_{\epsilon_m}^2 + \sigma_v^2 \sigma_{\epsilon_n}^2 + \sigma_v^2 \sigma_{\epsilon_m}^2)(\sigma_{\epsilon_n}^2 \sigma_{\eta_n}^2 + \sigma_v^2 \sigma_{\epsilon_n}^2 + \sigma_v^2 \sigma_{\eta_n}^2)^2}}{3C\psi_{r_n}^2 + B} > 0$$

The above derivative is positive as $1 > \frac{A}{B} > \psi_{r_n}^*$.

Part (vii): The expected bias decreases in σ_{η_n}:

$$= -\frac{\mu_\theta}{2c}\frac{\frac{\partial F(\psi_{r_n})}{\partial \sigma_{\eta_n}}}{3C\psi_{r_n}^2 + B} < 0 \tag{34}$$

The inequality in (34) follows because $\frac{\partial F(\psi_{r_n})}{\partial \sigma_{\eta_n}} > 0$ as $\frac{A}{B} > \psi_{r_n}^*$.

Value Relevance

We continue to show the proofs for value relevance:

$$VR = \frac{Var(v|x,y)}{Var(v|x,y,r_n^*)} = 1 + \frac{\sigma_v^2\sigma_{\epsilon_n}^2\psi_{r_n}^*}{\sigma_{\epsilon_n}^2\sigma_{\eta_n}^2 + \sigma_v^2\sigma_{\eta_n}^2 + \sigma_v^2\sigma_{\epsilon_n}^2(1-\psi_{r_n}^*)} \tag{35}$$

The second equation above follows from (31). For the following proofs we define the right side of (35) as the function $H(\sigma_{\epsilon_n}, \sigma_{\eta_n}, \sigma_v, \psi_{r_n}^*(\sigma_{\epsilon_n}, \sigma_{\eta_n}, \sigma_v, c, \sigma_\theta, \sigma_{\epsilon_m}))$.

Part (i)

$$\frac{\partial VR}{\partial c} = \frac{\partial H}{\partial \psi_{r_n}^*}\frac{\partial \psi_{r_n}^*}{\partial c} > 0$$

The inequality follows from $\frac{\partial H}{\partial \psi_{r_n}^*} > 0$ and $\frac{\partial \psi_{r_n}^*}{\partial c} > 0$.

Part (ii) Value relevance is independent of μ_θ.

Part (iii) The value relevance is increasing in σ_θ

$$\frac{\partial VR}{\partial \sigma_\theta} = \frac{\partial H}{\partial \psi_{r_n}^*}\frac{\partial \psi_{r_n}^*}{\partial \sigma_\theta} > 0$$

The inequality follows from $\frac{\partial H}{\partial \psi_{r_n}^*} > 0$ and $\frac{\partial \psi_{r_n}^*}{\partial \sigma_\theta} < 0$.

Part (iv): The value relevance is increasing in σ_v. The derivative is given by:

$$\frac{\partial VR}{\partial \sigma_v} = \frac{\partial H}{\partial \sigma_v} + \frac{\partial H}{\partial \psi_{r_n}^*}\frac{\partial \psi_{r_n}^*}{\partial \sigma_v} = \frac{\partial H}{\partial \sigma_v} - \frac{\partial H}{\partial \psi_{r_n}^*}\frac{\frac{\partial F(\psi_{r_n})}{\partial \sigma_v}}{\frac{\partial F(\psi_{r_n})}{\partial \psi_{r_n}}} > 0$$

To derive the inequality above. We substitute in the derivative $\frac{\partial F(\psi_{r_n})}{\partial \sigma_v}$ for $\psi^{*3}_{r_n} = \frac{A - B\psi^*_{r_n}}{C}$ which follows from (32). The above inequality follows then from $\frac{A}{B} > \psi^*_{r_n} > 0$.

Part (v): The value relevance is increasing in σ_{ϵ_n}. The derivative is given by:

$$\frac{\partial VR}{\partial \sigma_{\epsilon_n}} = \frac{\partial H}{\partial \sigma_{\epsilon_n}} + \frac{\partial H}{\partial \psi^*_{r_n}} \frac{\partial \psi^*_{r_n}}{\partial \sigma_{\epsilon_n}} = \frac{\partial H}{\partial \sigma_{\epsilon_n}} - \frac{\partial H}{\partial \psi^*_{r_n}} \frac{\frac{\partial F(\psi_{r_n})}{\partial \sigma_{\epsilon_n}}}{\frac{\partial F(\psi_{r_n})}{\partial \psi_{rn}}} > 0$$

To derive the inequality above. We substitute in the derivative $\frac{\partial F(\psi_{r_n})}{\partial \sigma_{\epsilon_n}}$ for $\psi^{*3}_{r_n} = \frac{A - B\psi^*_{r_n}}{C}$ which follows from (32). The above inequality follows then from $\frac{A}{B} > \psi^*_{r_n} > 0$.

Part (vi): Value relevance is increasing in σ_{ϵ_m}:

$$\frac{\partial VR}{\partial \sigma_{\epsilon_m}} = \frac{\partial H}{\partial \psi^*_{r_n}} \frac{\partial \psi^*_{r_n}}{\partial \sigma_{\epsilon_m}} > 0$$

as $\frac{\partial H}{\partial \psi^*_{r_n}} > 0$ and $\frac{\partial \psi^*_{r_n}}{\partial \sigma_{\epsilon_m}} > 0$.

Part (vii): The value relevance is decreasing in σ_{η_n}. The derivative is given by:

$$\frac{\partial VR}{\partial \sigma_{\eta_n}} = \frac{\partial H}{\partial \sigma_{\eta_n}} + \frac{\partial H}{\partial \psi^*_{r_n}} \frac{\partial \psi^*_{r_n}}{\partial \sigma_{\eta_n}} = \frac{\partial H}{\partial \sigma_{\eta_n}} - \frac{\partial H}{\partial \psi^*_{r_n}} \frac{\frac{\partial F(\psi_{r_n})}{\partial \sigma_{\eta_n}}}{\frac{\partial F(\psi_{r_n})}{\partial \psi_{r_n}}} < 0.$$

The inequality above follows from $\frac{\partial H}{\partial \sigma_{\eta_n}} < 0$ and $\frac{\partial F(\psi_{r_n})}{\partial \sigma_{\eta_n}} > 0$ as $\frac{A}{B} > \psi^*_{r_n}$.

Proof of Proposition 2.

To find the expected value of v given n and s_n, $E[v|n, s_n]$, we first need to determine its distribution function.

The multivariate normal distribution of v, n and s_n are given by:

$$f(v,n,s_n) = N(\begin{pmatrix} \mu_v \\ \mu_x \\ \mu_{s_n} \end{pmatrix}, \begin{pmatrix} \sigma_v^2 & COV_{nv} & 0 \\ COV_{nv} & \sigma_n^2 & COV_{ns_n} \\ 0 & COV_{ns_n} & \sigma_{s_n}^2 \end{pmatrix}) \tag{36}$$

where $\mu_x = \mu_v + \mu_{\epsilon_n}$, $\mu_{s_n} = \mu_{\epsilon_n}$, $COV_{ns_n} = \sigma_{\epsilon_n}^2$, $COV_{nv} = \sigma_v^2$ and $\sigma_{s_n}^2 = \sigma_{\epsilon_n}^2 + \sigma_{\eta_n}^2$.

Integrating $f(v,n,s_n)$ over v leads to the bivariate normal distribution of n and s_n.

$$f(n,s_n) = \int f(v,n,s_n)dv \tag{37}$$

The conditional density function of v given n and s_n is given by

$$f(v|n,s_n) = \frac{f(v,n,s_n)}{f(n,s_n)} \tag{38}$$

The conditional expected value of this distribution is then

$$E[v|n,s_n] = \int v \cdot f(v|n,s_n)\ dv = \mu_v + \frac{COV_{nv}}{\sigma_n^2} \cdot (\frac{1}{1-\rho_{ns_n}})(n - E[n|s_n]). \tag{39}$$

where $E[n|s_n] = \mu_x + \frac{Cov_{ns_n}}{\sigma_{s_n}^2}(s_n - \mu_{s_n})$ and ρ_{ns_n} is the correlation between n and s_n, $\rho_{ns_n} = \frac{Cov_{ns_n}}{\sigma_{s_n}\sigma_n}$. From (39) and $E[n|s_n]$ follows that the ratio of β_n and β_{s_n} is given by:

$$\frac{\beta_n}{\beta_{s_n}} = -\frac{\sigma_{s_n}^2}{Cov_{ns_n}} = -\frac{\sigma_{\epsilon_n}^2 + \sigma_{\eta_n}^2}{\sigma_{\epsilon_n}^2}.$$

The same can be done for the measurement approach using m instead of n, which then explains proposition 2.

Proof of Proposition 3

Expected bias of measurement approach x is smaller than the one from measurement approach y if $\psi^*_{r_x} < \psi^*_{r_y}$. This follows from the equation of the expected bias: $Eb_n = \frac{\mu_\theta * \psi^*_{rn}}{2c}$. From equation (35) follows that $VR_x < VR_y$ iff $\psi^*_{r_x} < \frac{\sigma^2_{\epsilon y}\left(\sigma^2_{\epsilon x}\sigma^2_{\eta x}+\sigma^2_v\sigma^2_{\epsilon x}+\sigma^2_v\sigma^2_{\eta x}\right)}{\sigma^2_{\epsilon x}\left(\sigma^2_{\epsilon y}\sigma^2_{\eta y}+\sigma^2_v\sigma^2_{\epsilon y}+\sigma^2_v\sigma^2_{\eta y}\right)}\psi^*_{r_y} = \frac{A_y}{A_x}\psi^*_{r_y}$

As shown above $F_x(\psi_r)$ and $F_y(\psi_r)$ are both increasing function. When setting $F_x(\psi_r) = F_y(\psi_r)$, we find that these functions cut only ones at $\psi_r = \frac{A_x - A_y}{B_x - B_y}$.

Suppose $\sigma^2_{\eta_x} < \sigma^2_{\eta_y}$ and $\frac{\sigma^2_{\epsilon x}}{\sigma^2_{\eta x}} \leq \frac{\sigma^2_{\epsilon y}}{\sigma^2_{\eta y}}$ and $\sigma^2_v < \frac{\sigma^2_{\epsilon x}\sigma^2_{\epsilon y}\left(\sigma^2_{\eta y}-\sigma^2_{\eta x}\right)}{\sigma^2_{\epsilon y}\sigma^2_{\eta x}-\sigma^2_{\epsilon x}\sigma^2_{\eta y}}$. It follows that $A_x > A_y$ and $A_x B_y > A_y B_x$. If $B_x < B_y$, then $F_x(\psi_r)$ and $F_y(\psi_r)$ cut at a $\psi_r < 0$ and hence, for all $\psi_r > 0$ we have the following $F_x(\psi_r) < F_y(\psi_r)$ and hence $\psi^*_{r_x} > \psi^*_{r_y}$. If $B_x > B_y$, then from $A_x B_y > A_y B_x$ it follows that $F_x\left(\frac{A_x - A_y}{B_x - B_y}\right) > 0$, hence, $F_x(\psi_r) < F_y(\psi_r)$ for all $\psi_r < \frac{A_x - A_y}{B_x - B_y}$, which implies $\psi^*_{r_x} > \psi^*_{r_y}$. In summary, for $\sigma^2_{\eta_x} < \sigma^2_{\eta_y}$, $\frac{\sigma^2_{\epsilon x}}{\sigma^2_{\eta x}} \leq \frac{\sigma^2_{\epsilon y}}{\sigma^2_{\eta y}}$ and $\sigma^2_v < \frac{\sigma^2_{\epsilon x}\sigma^2_{\epsilon y}\left(\sigma^2_{\eta y}-\sigma^2_{\eta x}\right)}{\sigma^2_{\epsilon y}\sigma^2_{\eta x}-\sigma^2_{\epsilon x}\sigma^2_{\eta y}}$ we have $\psi^*_{r_x} > \psi^*_{r_y} > \frac{A_y}{A_x}\psi^*_{r_y}$. The second inequality follows from $A_x > A_y$. Thus, we have $Eb_x > Eb_y$ and $VR_x > VR_y$.

Suppose, $\sigma^2_{\eta_x} < \sigma^2_{\eta_y}$, $\frac{\sigma^2_{\epsilon x}}{\sigma^2_{\eta x}} \leq \frac{\sigma^2_{\epsilon y}}{\sigma^2_{\eta y}}$ and $\sigma^2_v \geq \frac{\sigma^2_{\epsilon x}\sigma^2_{\epsilon y}\left(\sigma^2_{\eta y}-\sigma^2_{\eta x}\right)}{\sigma^2_{\epsilon y}\sigma^2_{\eta x}-\sigma^2_{\epsilon x}\sigma^2_{\eta y}}$ from this inequalities it follows that $A_x < A_y$ and $B_x < B_y$. Hence, $\psi_r = \frac{A_x - A_y}{B_x - B_y}$ (remember that $F_x\left(\frac{A_x - A_y}{B_x - B_y}\right) = F_y\left(\frac{A_x - A_y}{B_x - B_y}\right)$) is larger than zero. If functions F_x and F_y cut at a value smaller than zero ($F_x\left(\frac{A_x - A_y}{B_x - B_y}\right) < 0$) then $\psi^*_{r_x} > \psi^*_{r_y}$. Inequality $F_x\left(\frac{A_x - A_y}{B_x - B_y}\right) < 0$ holds for all $g \equiv \frac{c}{\sigma_\theta} > \bar{\bar{g}}$. $\bar{\bar{g}}$ is the unique solution of equation $F_x\left(\frac{A_x - A_y}{B_x - B_y}\right) = 0$. If $g \equiv \frac{c}{\sigma_\theta} < \bar{\bar{g}}$ then if $\psi^*_{r_x} < \psi^*_{r_y}$ from $A_x < A_y$ follows that $\psi^*_{r_x} < \frac{A_y}{A_x}\psi^*_{r_y}$ and hence $VR_x < VR_y$. Hence, there is no region where x dominates. If $\bar{\bar{\bar{g}}} > g \equiv \frac{c}{\sigma_\theta} > \bar{\bar{g}}$ then $VR_x < VR_y$ and if $g > \bar{\bar{\bar{g}}}$ then we have $VR_x > VR_y$ where $\bar{\bar{\bar{g}}}$ solves the system of equations $F_x(\psi^*_x)$, $F_y\left(\psi^*_y\right)$ and $\psi^*_{r_x} = \frac{A_y}{A_x}\psi^*_{r_y}$. $\bar{\bar{\bar{g}}} = \bar{\bar{g}} = 0$ for $\sigma^2_v = \frac{\sigma^2_{\epsilon x}\sigma^2_{\epsilon y}\left(\sigma^2_{\eta y}-\sigma^2_{\eta x}\right)}{\sigma^2_{\epsilon y}\sigma^2_{\eta x}-\sigma^2_{\epsilon x}\sigma^2_{\eta y}}$ as $A_x = A_y$ and $\bar{\bar{\bar{g}}} > \bar{\bar{g}}$ for all $\sigma^2_v > \frac{\sigma^2_{\epsilon x}\sigma^2_{\epsilon y}\left(\sigma^2_{\eta y}-\sigma^2_{\eta x}\right)}{\sigma^2_{\epsilon y}\sigma^2_{\eta x}-\sigma^2_{\epsilon x}\sigma^2_{\eta y}}$.

If $\sigma^2_{\eta_x} > \sigma^2_{\eta_y}$ and $\frac{\sigma^2_{\epsilon x}}{\sigma^2_{\eta x}} \leq \frac{\sigma^2_{\epsilon y}}{\sigma^2_{\eta y}}$ then $A_x < A_y$ and hence there exists no region IV as if $\psi^*_{r_x} < \psi^*_{r_y}$ then because of $A_x < A_y$ it follows that $\psi^*_{r_x} < \frac{A_y}{A_x}\psi^*_{r_y}$ and hence $VR_x < VR_y$.

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